

5.73

Quiz 23 ANSWERS

$$\mathbf{J}^2 |JM\rangle = \hbar^2 J(J+1) |JM\rangle$$

$$\mathbf{J}_z |JM\rangle = \hbar M |JM\rangle$$

$$\mathbf{J}_\pm = \mathbf{J}_x \pm i\mathbf{J}_y$$

$$\mathbf{J}_\pm |JM\rangle = [J(J+1) - M(M \pm 1)]^{1/2} |JM \pm 1\rangle$$

A. What are the ΔJ and ΔM selection rules for the following operators:

(i) \mathbf{J}^4 $\Delta J = 0, \Delta M = 0$

(ii) $(\mathbf{J}_+)^2$ $\Delta J = 0, \Delta M = +2$

(iii) $\mathbf{J}_+\mathbf{J}_-$ $\Delta J = 0, \Delta M = 0$

(iv) \mathbf{J}_x $\Delta J = 0, \Delta M = \pm 1$

(v) $\bar{\mathbf{J}}$ $\Delta J = 0, \Delta M = (\hat{i}(\pm 1), \hat{j}(\pm 1), (\hat{k}0))$

B. What are the values of the following matrix elements:

(i) $\langle JM+1 | \mathbf{J}^2 | JM \rangle = 0$

(ii) $\langle JM | \mathbf{J}^2 \mathbf{J}_z | JM \rangle = \hbar^3 J(J+1)M$

(iii) $\langle JM | \mathbf{J}_+ \mathbf{J}_- | JM \rangle = \hbar^2 ([J(J+1) - M(M+1)][J(J+1) - M(M-1)])^{1/2}$

(iv) $\langle JM | \mathbf{J}_+ \mathbf{J}_- - \mathbf{J}_- \mathbf{J}_+ | JM \rangle = 0$

(v) $\langle JM+1 | \mathbf{J}_x | JM \rangle$

$$= \left\langle JM+1 \left| \frac{1}{2} (\mathbf{J}_+ + \mathbf{J}_-) \right| JM \right\rangle = \frac{1}{2} \hbar [J(J+1) - M(M+1)]^{1/2}$$

C. What is the value of the commutator $[\mathbf{J}_+, \mathbf{J}_-] = ?$

$$[\mathbf{J}_+, \mathbf{J}_-] = [\mathbf{J}_x + i\mathbf{J}_y, \mathbf{J}_x - i\mathbf{J}_y] = i\hbar [0 - iJ_z - iJ_z + 0] = 2\hbar J_z$$

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