Problem Set 6

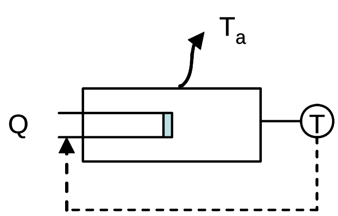
A metal block is to be kept at temperature T_{sp}. The value of T_{sp} is occasionally changed. A check of the Biot number convinces you that the temperature will be spatially uniform. Furthermore, you find that heat loss, over the range of expected operating temperatures, is adequately described by a linear relationship

$$Q_L = UA(T - T_a)$$

where T is the block temperature, T_a is the environment temperature, and UA is what you would expect it is. You embed a small resistance heater in the block; its power is linearly adjustable between 0 and Q_{max}. You also acquire a temperature sensor with adjustable bias and sensitivity

$$V_s = b + \beta T$$

where V_s is the sensor reading in mV for temperature T. You set b and β to obtain 0 mV at T_{low} and 10 mV at T_{high}. Finally you obtain a PI controller; Fran the electrician sets it up to accept a 10 mV input range and to output a signal that drives the heater between its limits. The controller settings are gain K_c^* (%out $\%in^{-1}$) and integral time T_I (s).



(1) Write equations for each of the four components of the feedback loop. Combine these equations into a single equation for the closed loop response of T to T_a and set point T_{sp}. It will be an integro-differential equation; arrange it so that all instances of T are on the left-hand side. In your equations, use physical variables; that is, every variable and parameter should be some observable or settable quantity. Please employ the nomenclature used above, and introduce other physical quantities as you need them.

(2) Define a reference condition and recast the component equations in terms of deviation variables. Use Laplace transforms to define transfer functions for the components. Combine the transfer functions into a closed loop model that gives the response of T'(s) to T_a'(s) and set point T_{sp}'(s). Present these transfer functions in terms of appropriate gains and time constants, which are defined in terms of the physical parameters.

(3) Recast the component equations in terms of 0-100% scaled variables. Combine them into a single equation for the closed loop response of T^* to T_a^* and set point T_{sp}^*. Define range limits as needed. Don't omit factors of "100%" if they are part of the units conversion.

In each of these presentations, scrutinize your results for dimensional consistency. Pay particular attention to gains, scaling factors, conversion factors, etc. There should be no dimensional discrepancies. Also, don't forget the bias terms: with no error, only deviation variables are zero.