$$g(\vec{x}) = \int k(\vec{x}-\vec{x}')\cdot f^{\circ}(\vec{x})\,d\vec{x}' + W(\vec{x})$$

$(f^{\circ}, k) \rightarrow$ provides infinite number of possible solutions

What types of constraints apply?

- non-negativity
- symmetry on k
- frequency space expected for object
- support for PSF $\rightarrow$ zero beyond some distance
- maximum intensity for object

Noiseless: $g^{\circ}(\vec{x}) = \int k(\vec{x}-\vec{x}')f^{\circ}(\vec{x})\,d\vec{x}'$

$$C_g \equiv \{(u,v) : \underbrace{u * v}_{\text{convolution}} = g^{\circ}\} \rightarrow (f^{\circ}, k) \in C_g$$

$C_f \equiv \{(u,v)$: u satisfies constraints on f$\}$

$C_k \equiv \{(u,v)$: v satisfies constraints on k$\}$

Let $C_o = C_g \cap C_f \cap C_k$

To find a solution in C_o:

Iterative projections:

Let P_g be an operator that projects into C_g

& P_f operator projects into C_f

& P_k operator projects into C_k

$$(f,k)_{i+1} = P_k P_f P_g (f,k)_i$$

initial guess $(f,k)_o$

$\rightarrow$ iterations converge to point in C_o

Projection operator: often a form of minimization

minimize $\|(u,v) - (f', k')\|$ subject to constraints

solution ↑ ↑ operand; applied with Lagrange multipliers ↑ $(u,v) \in C_g$ or C_f or C_k

Let $J_{(u,v)} \equiv \|g^{\circ} - u * v\|^2 = \int [g^{\circ}(\vec{x}') - (u*v)(\vec{x}')]^2 d\vec{x}' \geq 0$

measure of error to measured image

$(u,v) \in C_g \Rightarrow J_{(u,v)} = 0$

C_f & C_k

1) initial guess: (f_o, k_o)

$f_o = P_f g^{\circ}$, $[k_o = 0]$

2) Solve for $k_i = \arg\left\{\min_{k \in C_k} J_{(f_{i-1}, k)}\right\}$

3) Solve for $f_i = \arg\left\{\min_{f \in C_f} J_{(f, k_i)}\right\}$

4) increment $i \rightarrow i+1$

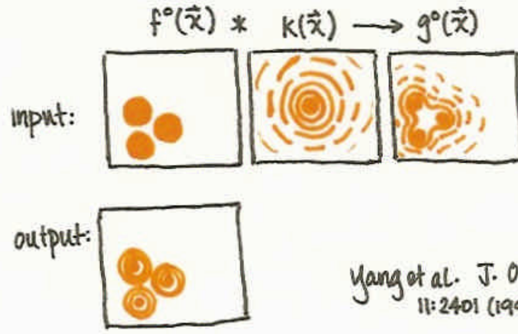
Poorer Alternative

$$E = J_{(f,k)} + E_{restraint} \quad k(\vec{x}) = 0 \text{ for } \Omega_k$$

$$E_{restraint} = \int_{\Omega_f} |f(\vec{x})|^2 d\vec{x} + \int_{\Omega_k} |k(\vec{x})|^2 d\vec{x}$$

$g(\vec{x})$ is invariant to $(f,k) \rightarrow (\alpha f, \frac{k}{\alpha})$

SAMPLE PROBLEM:



Yang et al. J. Opt. Soc. Am A 11:2401 (1994)