

# **Brownian motion and mean-squared displacement**

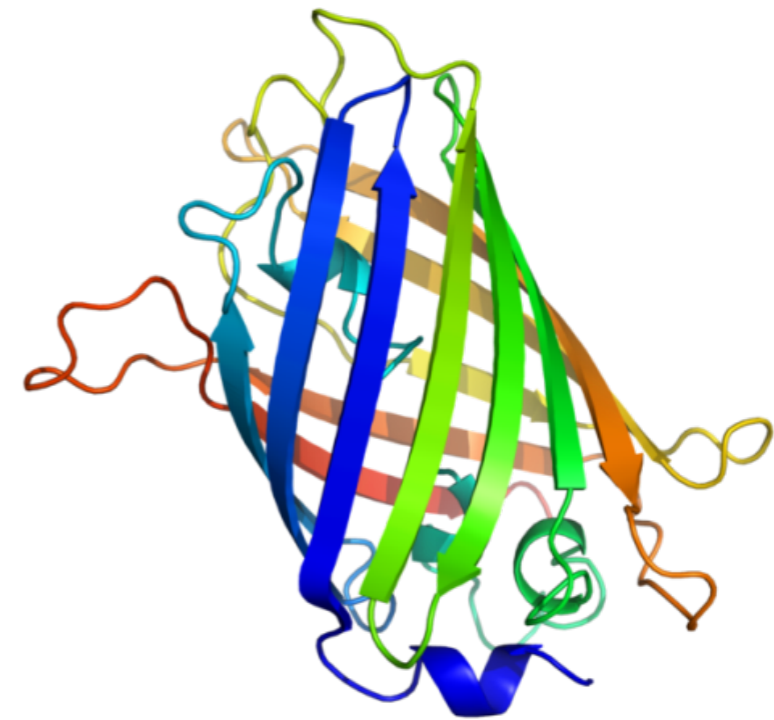
# The discovery of Green Fluorescent Protein (GFP) has revolutionized *in vivo* biology

*Aequorea victoria*



Courtesy of [Sierra Blakely](#). Used with permission.

GFP



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Extraction, purification and properties of Aequorin, a bioluminescent protein from the Luminous Hydromedusan, *Aequorea*.

Shimomura, Johnson, and Saiga  
*J Cell Comp Physiol* 59: 223 (1962)

# Prasher and Chalfie demonstrated GFP can be expressed in bacteria and is still fluorescent

gfp10 cDNA  
GeneBank Accession number M62653

```

1  tacacacgaa taaaagataa caaagatgag taaaggagaa gaacttttca ctggagttgt
61  cccaattctt gttgaattag atggtgatgt taatgggcac aaatcttctg tcagtggaga
121 gggatgaagg gatgcaacat acggaaaact tacccttaaa tttatcttgc ctactggaaa
181 actacctgtt ccatggccaa cacttgctac tactttctct tatggtgttc aatgcttttc
241 aagatacca gatcatatga aacagcatga ctttttcaag agtgccatgc ccgaaggtta
301 tgtacaggaa agaactatat ttttcaaaga tgacgggaac tacaagacac gtgctgaagt
361 caagtttgaa ggtgatcccc ttgttaatag aatcgagtta aaaggtattg attttaaaga
421 agatggaaac attcttgac acaaattgga atacaactat aactcacaca atgtatacat
481 catggcagac aaacaaaaga atggaatcaa agttaacttc aaaattagac acaacattga
541 agatggaagc gttcaactag cagaccatta tcaacaaaat actccaattg gcgatggccc
601 tgccttttta ccagacaacc attacctgtc cacacaatct gccctttcga aagatcccaa
661 cgaaaagaga gaccacatgg tccttcttga gtttgtaaca gctgctggga ttacacatgg
721 catggatgaa ctatacaaat aaatgtccag acttccaatt gacactaaag tgtccgaaca
781 attactaaaa tctcagggtt cctgggttaa ttcaggctga gatattattt atatatttat
841 agattcatta aaattgtatg aataatttat tgatgttatt gatagagggt attttcttat
901 taaacaggct acttgagtg tattcttaat tctatattaa ttacaatttg atttgacttg
961 ctcaaa

```

Primary structure of the *Aequorea victoria* green-fluorescent protein. Prasher, Eckenrode, Ward, Prendergast, and Cormier, *Gene* **111**:229 (1992)

**Fig. 1.** Expression of GFP in *E. coli*. The bacteria on the right side of the figure have the GFP expression plasmid. Cells were photographed during irradiation with a hand-held long-wave UV source.



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Green fluorescent protein as a marker for gene expression. Chalfie, Tu, Euskirchen, Ward, and Prasher *Science* **263**: 802 (1994)

# Brownian Motion inside Cells: Super-resolution imaging of ribosomes in a living *E. coli*

Figures removed due to copyright restrictions.

Source: Bakshi, Somenath et al. "[Superresolution imaging of ribosomes and RNA polymerase in live Escherichia coli cells.](#)" *Molecular Microbiology* 85, no. 1 (2012): 21-38.

# Measuring diffusivity using single-particle tracking

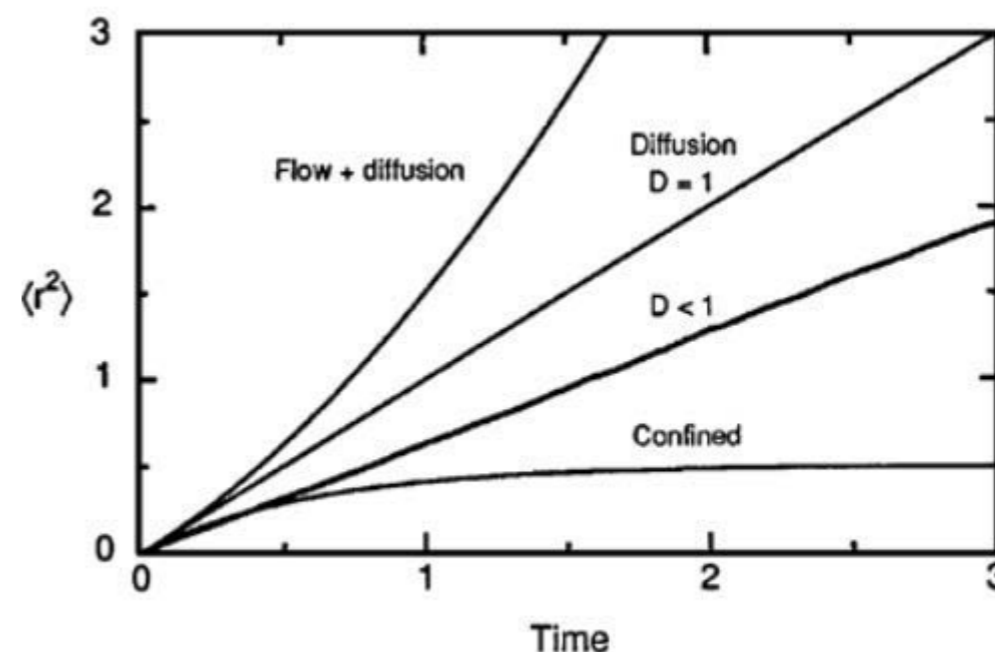
## Single-particle Tracking: Application to Membrane Dynamics

Saxton & Jacobson

*Annu Rev Biophys Biomol Struct* 1997

$$\text{MSD}(\tau) \equiv \langle \Delta \mathbf{r}(\tau)^2 \rangle = \frac{1}{N - \tau} \sum_{i=1}^{N-\tau} |\mathbf{r}_{i+\tau} - \mathbf{r}_i|^2$$

Mean-square displacement versus time for diffusion + flow, pure normal diffusion, sub-diffusion, and confined diffusion



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$$\langle r^2 \rangle = 4Dt$$

normal diffusion

$$\langle r^2 \rangle = 4Dt^\alpha$$

anomalous diffusion

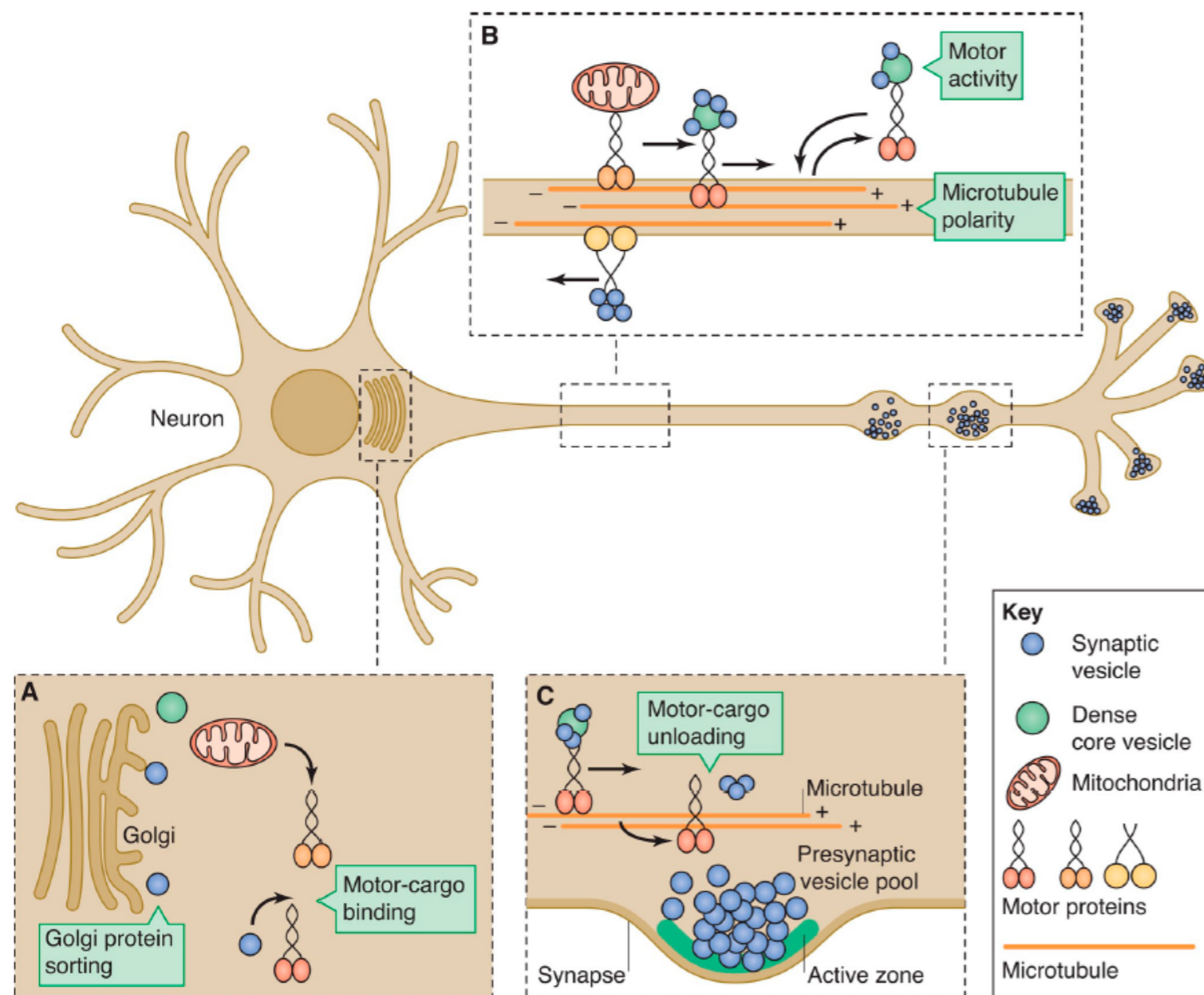
$$\langle r^2 \rangle = 4Dt + (Vt)^2$$

directed motion with diffusion

$$\langle r^2 \rangle \simeq \langle r_C^2 \rangle [1 - A_1 \exp(-4A_2Dt / \langle r_C^2 \rangle)]$$

corralled motion

# Long-range neuronal transport mediated by molecular motors



Courtesy of the authors. Used with permission.  
 Source: Chia, Poh Hui et al. "Cellular and molecular mechanisms underlying presynapse formation." *The Journal of Cell Biology* 203, no. 1 (2013): 11-22.

Chia et al., *J Cell Biol* 2013

Table 8-1 removed due to copyright restrictions.  
 Source: Brady, Scott et al, eds. *Basic neurochemistry: principles of molecular, cellular, and medical neurobiology*. Academic Press, 2011.

Table from Scott Brady


Velocity ~1 microns/second via motor transport but only ~1-10% of time is spent actively transported, thus 16 minutes from L/V becomes order of a day

# Fick's First Law

# Fick's First Law

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**Adolf Fick**



Adolf Eugen Fick (1829-1901)

<b>Born</b>	3 September 1829 Kassel, Electorate of Hesse
<b>Died</b>	21 August 1901 (aged 71) Blankenberge, Flanders
<b>Nationality</b>	German
<b>Fields</b>	Physiology Biophysics
<b>Institutions</b>	University of Zurich University of Würzburg
<b>Alma mater</b>	University of Marburg
<b>Doctoral advisor</b>	Franz Ludwig Fick <sup>[1]</sup>

Source: "[Adolf Eugen Fick](#)" article on Wikipedia.



# Derivation of Fick's First Law

Book cover removed due to copyright restrictions.  
Source: Grodzinsky, Alan. *Field, Forces, and Flows in Biological Systems*. Garland Science, 2011.

Figure 1.3 and Equations 1.23-1.26 removed due to copyright restrictions.  
Source: Grodzinsky, Alan. *Field, Forces and Flows in Biological Systems*.  
Garland Science, 2011. [Preview with [Google Books](#)]

# **Stokes-Einstein relation for diffusivity**

**Proteins, drugs, viruses, bacteria, etc. are all modeled as spheres with effective radius,  $a$ ,  $r$ ,  $R_0$ , etc.**

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## Sir George Stokes, Bt.

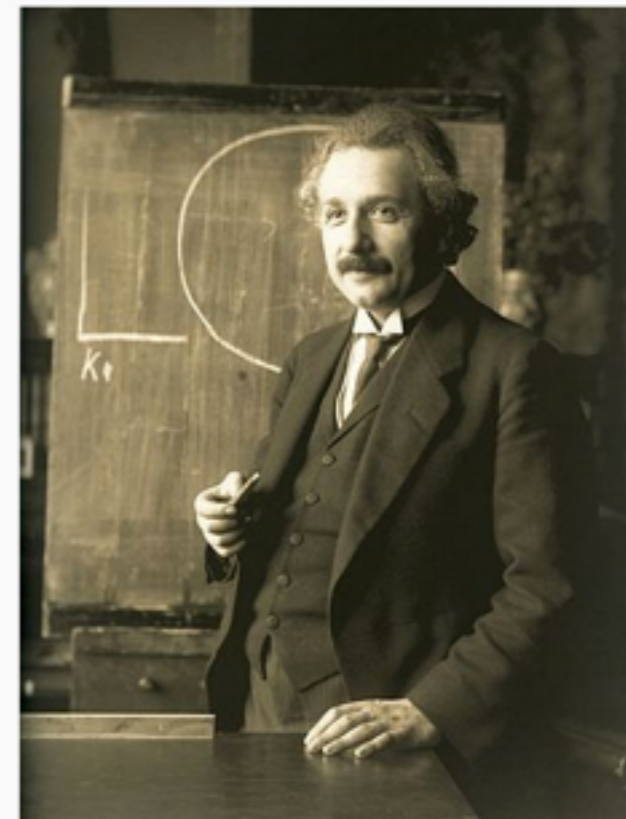


Text from "[Stoke's law](#)" article on wikipedia removed due to copyright restrictions.

<b>Born</b>	13 August 1819 <a href="#">Skreen, County Sligo, Ireland</a>
<b>Died</b>	1 February 1903 (aged 83) <a href="#">Cambridge, England</a>
<b>Fields</b>	Mathematics and <a href="#">physics</a>
<b>Institutions</b>	University of Cambridge
<b>Alma mater</b>	<a href="#">Pembroke College, Cambridge</a>

Source: "[Sir George Stokes](#)" article on Wikipedia.

## Albert Einstein

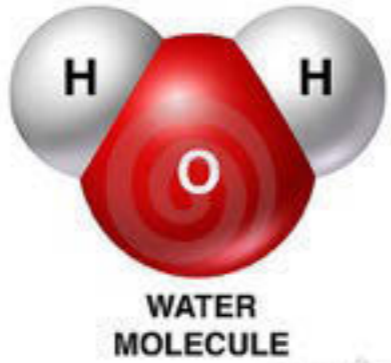
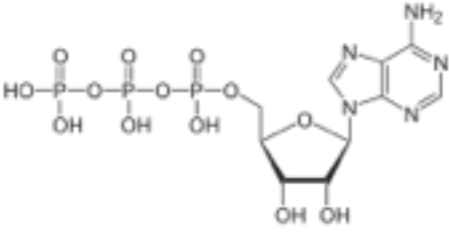
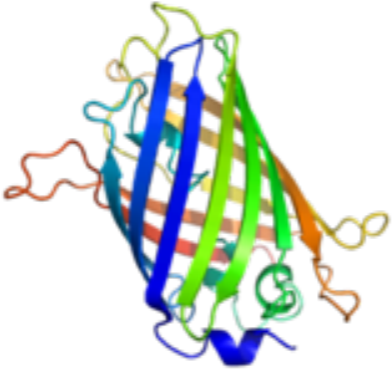


Albert Einstein in 1921

Text from "[Einstein relation \(kinetic theory\)](#)" article on wikipedia removed due to copyright restrictions.

<b>Born</b>	14 March 1879 <a href="#">Ulm, Kingdom of Württemberg, German Empire</a>
<b>Died</b>	18 April 1955 (aged 76) <a href="#">Princeton, New Jersey, United States</a>
<b>Residence</b>	Germany, Italy, Switzerland, Austria (today: Czech Republic), Belgium, United States
<b>Citizenship</b>	<a href="#">Kingdom of Württemberg</a> (1879–1896) <a href="#">Stateless</a> (1896–1901) <a href="#">Switzerland</a> (1901–1955) <a href="#">Austria of the Austro-Hungarian Empire</a> (1911–1912) <a href="#">Germany</a> (1914–1933) <a href="#">United States</a> (1940–1955)

# Estimating diffusivity using Stokes-Einstein

Molecular diffusivities in water at RT		MW [g/mol or Daltons]	a [nm]	D [ $\mu\text{m}^2/\text{sec}$ ] estimated (measured)
H <sub>2</sub> O	 <p>WATER MOLECULE</p>	18	0.2	1,500 in water (2,300 in water)
ATP		507	0.6	314 in water (150 in cytoplasm)
GFP		27,000	2.2	100 in water (20 in cytoplasm)

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Fall 2015

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