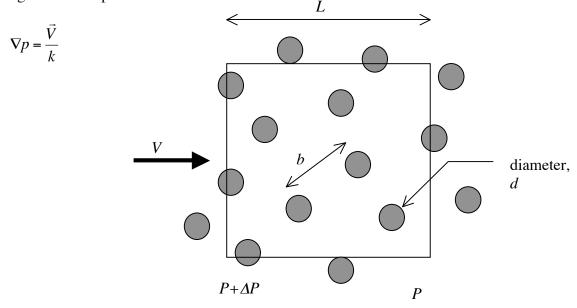**Fiber matrix permeability**

Consider the flow of liquid through a fibrous matrix such as the cytoskeleton or extracellular matrix. To a first approximation, this situation can be modeled as flow through a collection of rigid cylinders of circular cross-section (representing the macromolecular constituents of the extracellular matrix or the fibers of the cytoskeleton) surrounded by liquid with a viscosity equal to that of water. The objective of this problem is to obtain an estimate the "hydraulic permeability" of this matrix.

a) First, estimate using a scaling analysis how the drag force per unit length acting on one isolated fiber scales with the velocity of flow perpendicular to its axis (V), the fiber diameter (d) and the viscosity of the fluid (μ), on the assumption that inertial effects can be neglected[1].

b) Now consider a matrix comprised of identical, parallel fibers. Given that the average spacing between fibers is b, obtain a scaling relation for the pressure drop due to flow past a collection of fibers over a distance L where $L<<b$. (Hint: Consider a force balance on the fluid contained in a cube of volume L^3.)

Note that the *hydraulic permeability*, k, is defined by the following expression for the pressure gradient in a porous medium:

$$\nabla p = \frac{\vec{V}}{k}$$

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[1] Strictly speaking, inertia cannot be neglected in this situation since the inertia of the fluid at large distances from the cylinder eventually become important. See e.g., Lamb, *Hydrodynamics*, p. 614 for a detailed account of this point.

Fiber matrix permeability - Solution

a) The force acting on the fibers is due to fluid shear stress and the pressure distribution due to shear. If the fiber solid fraction is sufficiently small, then the fibers act as isolated cylinders and the length scale over which the velocity varies should be proportional to the fiber diameter, d. Shear stress (and the pressure distribution associated with it) therefore scales as $\mu V/d$ and the force per unit length of fiber (f) is this times a measure of the circumference, d or:

$$f \approx \left(\frac{\mu V}{d}\right) d \approx \mu V$$

Interestingly, this result is independent of fiber diameter, d. This can also be obtained by direct use of dimensional analysis. Recognizing that the force per unit length is a function of the parameters V, d and μ, it can be seen that a dimensionless force must equal a constant. This leads to the expression:

$$\frac{f}{\mu V} = const$$

leading to the same result as above.

b) If we let N be the number of fibers contained in a cube of volume L^3, then $N \approx (L/b)^2$.

A force balance on the fluid contained in a cube L on each side gives:

$$\Delta P L^2 \approx NfL \approx \left(\frac{L}{b}\right)^2 \mu V L$$

or

$$\frac{\Delta P}{L} \approx \nabla P \approx \frac{\mu V}{b^2}$$

The hydraulic permeability then scales according to $k \propto b^2/\mu$.

L

V

b

diameter, d

$P+\Delta P$

P