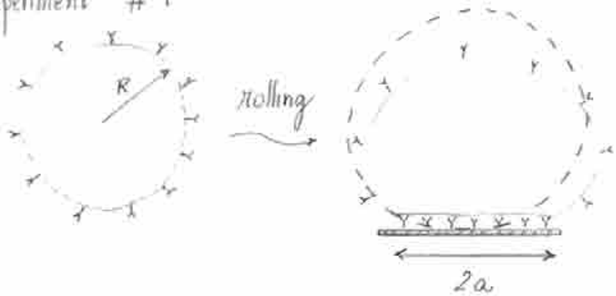
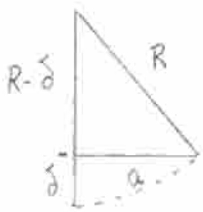
- **Adhesion** scaling relationships
  - experiment # 1



initial configuration - - - R : radius of the cell
after adhesion ——— δ : vertical height change due to flattening

how are R, δ and a related ?



$$R^2 = a^2 + (R-\delta)^2 = a^2 + R^2 - 2R\delta + \delta^2$$

$$\boxed{\delta \sim \frac{a^2}{R}}$$

analysis : idea " it takes energy to deform an elastic body, which is due to adhesion energy .

- elastic energy $\sim E\,\varepsilon^2\, dV$
- adhesion energy $\sim J a^2$

E Young's modulus
dV scales as a^3 (volume of deformation)
ε strain $\sim \frac{\delta}{a}$
J adhesion energy / unit area
energy of receptor - ligand interaction
* number of molecules per area

▷ balance elastic ~ adhesion

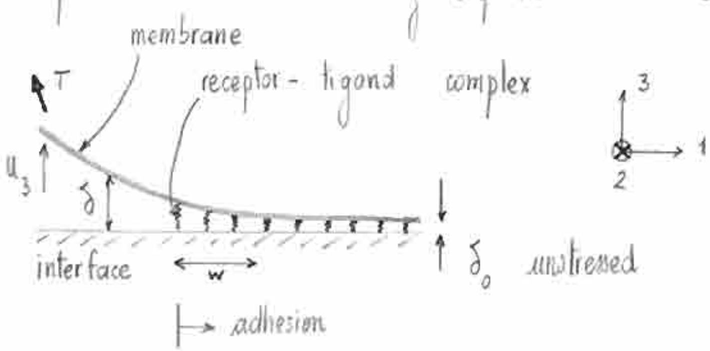
$$E\,\delta^2 a \sim E\,\frac{a^5}{R^2} \sim J a^2$$

$$\boxed{J \sim E\,\frac{a^3}{R^2}} \quad \text{or} \quad \boxed{a \sim \left(\frac{J}{E}\right)^{1/3} R^{2/3}}$$

more rigorous (with prefactors !) : JKR theory
static experiment !

$$a = \left(\frac{9\pi\, J\, R^2 (1-\nu^2)}{2E}\right)^{1/3}$$

- experiment # 2 Peeling of a membrane



Question : in what region are complexes stressed ?
w ?

Assume : bending dominates ⇒ pure bending

$$p = K_B\,\frac{\partial^4 u_3}{\partial x_1^4}$$

↳ due to pressure stress of bond stretching

$p \sim N_c\, f$ with N_c # complexes / area
f force per bond

Assume $f \sim k(\delta - \delta_0)$, k : spring constant

$f \sim k\, u_3$

hence
$$K_B \frac{\partial^4 u_3}{\partial x_1^4} \sim N_c\, k\, u_3$$

$$K_B \frac{u_3}{w^4} \sim N_c\, k\, u_3 \quad \text{or} \quad \boxed{w \sim \left(\frac{K_B}{N_c\, k}\right)^{1/4}}$$

localized contribution of complexes in region w

- Dynamics vs equilibrium

- affinity :

  R ≡ receptors
  L ≡ ligands
  C ≡ complexes

$$[R] + [L] \underset{k_-^0}{\overset{k_+^0}{\rightleftharpoons}} [C]$$

affinity $K^0 = \dfrac{k_+^0}{k_-^0}$ "0" : no force applied

in 2D $[R] \sim \mu m^{-2}$

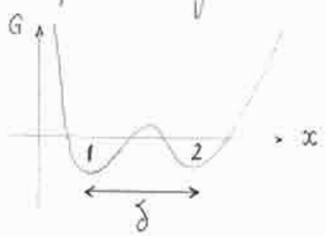
$k_+^0 \sim \mu m^{-2}\, s^{-1}$

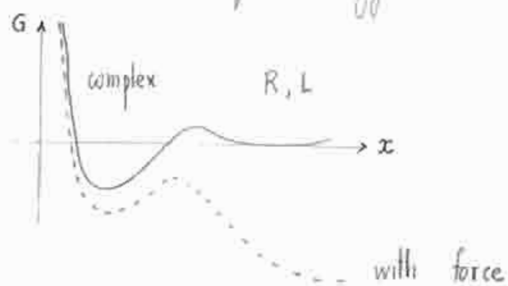
$k_-^0 \sim s^{-1}$

- how does force change affinity ? force changes state of equilibrium , tilts free energy coordinate

$$G \simeq G^0 - F x$$

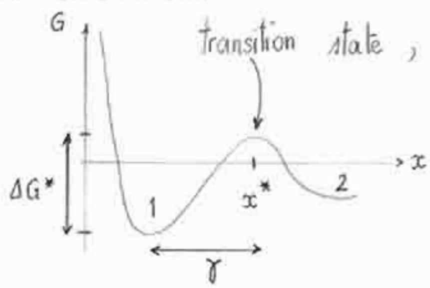
$$K^F = K^0 \exp\left(\frac{F\delta}{k_B T}\right)$$





- transition state

transition state , or activated state $[E_A]$



Using Boltzmann $\dfrac{[E_A]}{[E_1]} = \exp\left(\dfrac{-\Delta G^*}{k_B T}\right)$

Breakdown of E_A at rate A

▷ rate of formation of product 2 : $k_+ = A \exp\left(\dfrac{-\Delta G^*}{k_B T}\right)$

- The Bell model for adhesion

$$[R] + [L] \underset{k_-}{\overset{k_+}{\rightleftharpoons}} [C]$$

$$k_- = k_-^0 \exp\left(\frac{\gamma F}{k_B T}\right)$$

$\begin{cases} k_-^0 & \text{without force} \\ \gamma & \text{relative to transition state ; responsiveness of bond to force.} \end{cases}$