

Finite Volume in 2D

$$\frac{d}{dt} \int_{\Omega} P dx = - \int_{\partial\Omega} \vec{n} \cdot \vec{F}(P) ds$$

$$\frac{d}{dt} \int_L^R P dx = \vec{F}(P(L)) - \vec{F}(P(R))$$

$$\bar{P}_k = \frac{\int_L^R P dx}{\Delta x_k}$$

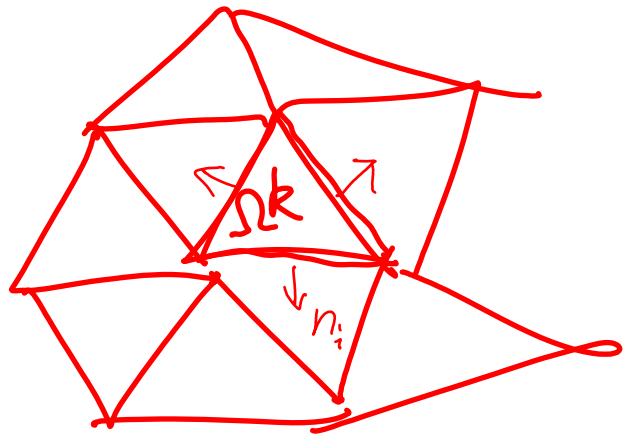
$$\frac{d}{dt} \bar{P}_k = \frac{1}{\Delta x_k} (\vec{F}_{k-\frac{1}{2}} - \vec{F}_{k+\frac{1}{2}})$$

$$\bar{P}_k = \frac{\int_{\Omega_k} P dx}{A_k}$$

$$\frac{d\bar{P}_k}{dt} = \frac{1}{A_k} \frac{d}{dt} \int_{\Omega_k} P dx$$

$$= -\frac{1}{A_k} \int_{\partial\Omega_k} \vec{n} \cdot \vec{F} ds$$

$$= -\frac{1}{A_k} \sum_{i \in S_k} \vec{n}_i \cdot \vec{F}_i S_i$$



turn PDE \rightarrow ODE $F(\bar{P}_k - \bar{P}_{\text{neighbor}})$

$$\frac{d}{dt} \begin{pmatrix} \bar{P}_1 \\ \bar{P}_2 \\ \vdots \\ \bar{P}_N \end{pmatrix} = F \begin{pmatrix} \bar{P}_1 \\ \vdots \\ \bar{P}_N \end{pmatrix}$$

Order of accuracy

$$\frac{du}{dt} = f(u)$$

$$\left[\frac{\delta}{\delta t} \right] u - f(u) \neq 0$$

$$\text{FE} \quad \frac{u^{k+1} - u^k}{\Delta t} - f(u^k) \neq 0$$
$$= O(\Delta t^P)$$

P is local order

$$\tau = u^{k+1} - (u^k + \Delta t f(u^k)) = O(\Delta t^{P+1})$$

Local

Global = Local

iff zero stable

under the $p \geq 1$ local.

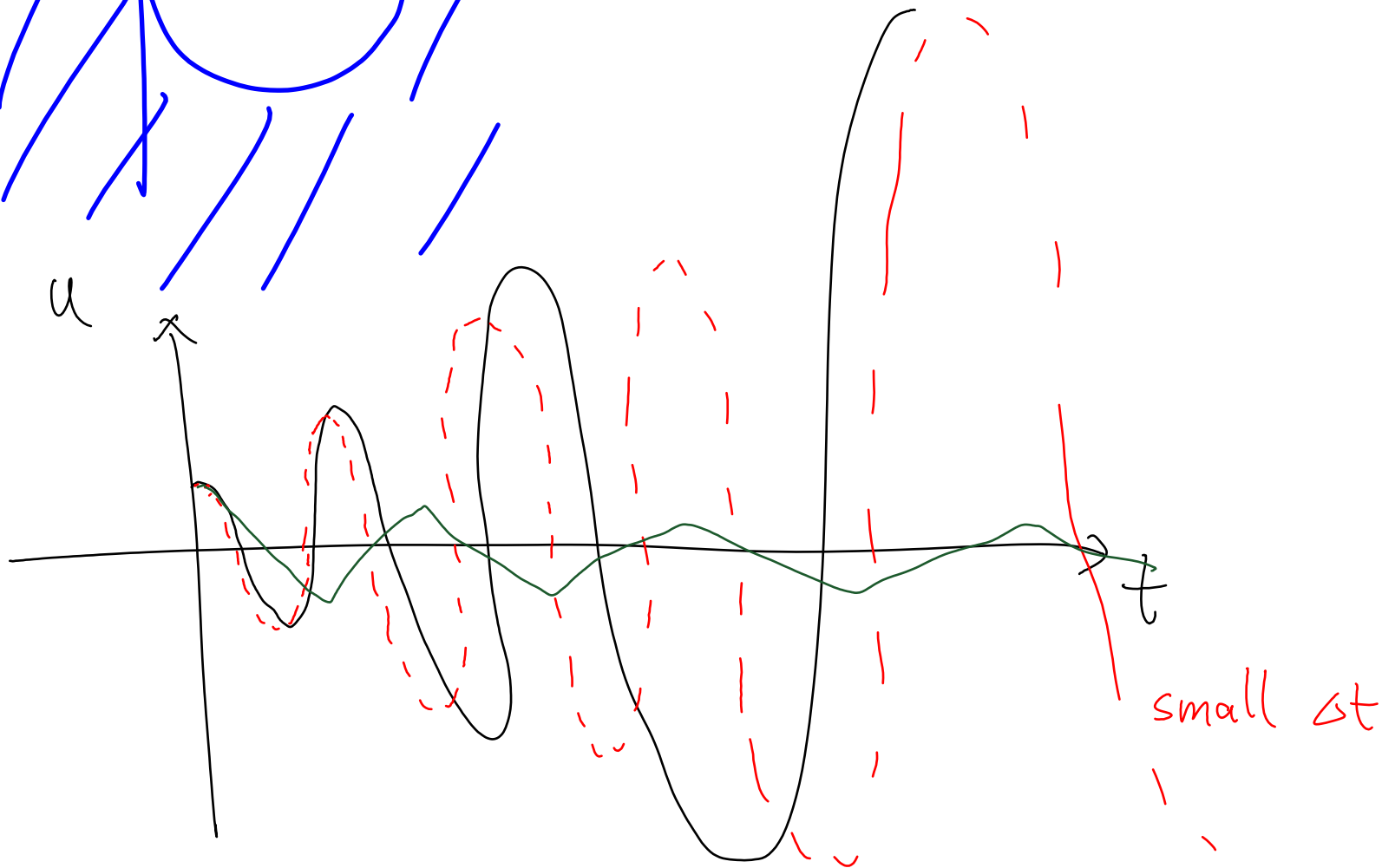
Eigenvalue stability

V^k is bounded for $\frac{dy}{dt} = \lambda y$ eigen

$\frac{dy}{dt} = 0$ zero

Scheme $(\lambda \Delta t)$

↓ ↓



Stiffness and Newton-Raphson

Implicit

Explicit

Lots of coding
Solve nonlinear Eqn

Larger stability region
in stiff problems

$$F(u) = 0$$

$$\frac{u - u^k}{\Delta t} = f(u, u^k, \dots)$$

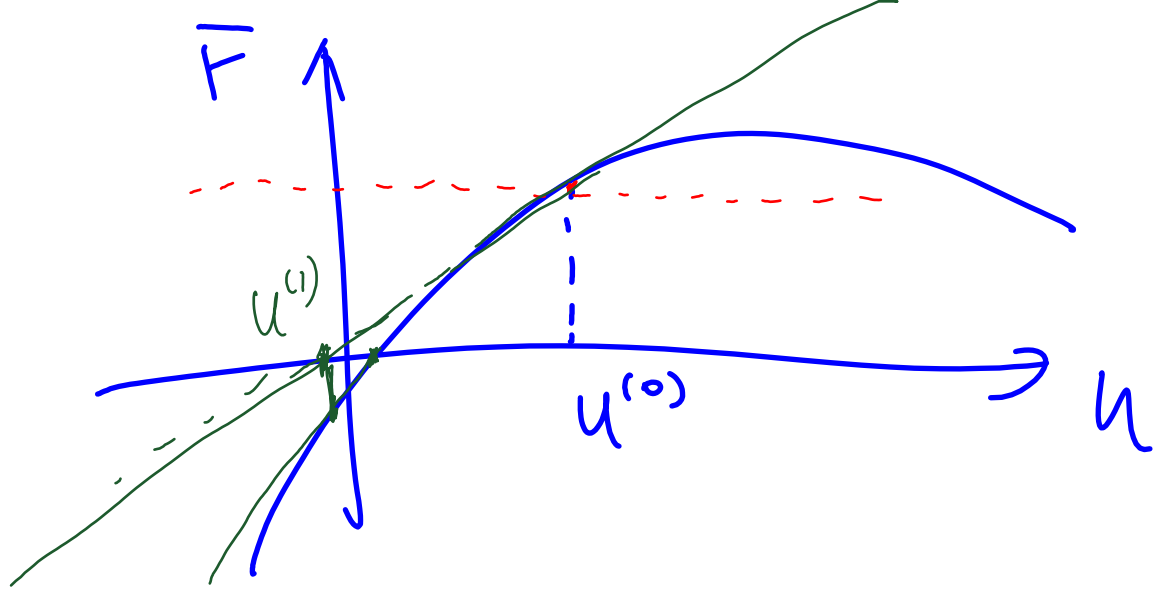
$$F: \frac{u - u^k}{\Delta t} - f(u, u^k, \dots)$$

$$u^{(0)} = u^k$$

$$\left. \begin{array}{l} F_1(u) \approx F_1(u^{(0)}) + \sum_{i=1}^N \frac{\partial \bar{F}_1}{\partial u_i} (u_i - u_i^{(0)}) \\ \vdots \\ F_N(u) \approx F_N(u^{(0)}) + \sum_{i=1}^N \frac{\partial \bar{F}_N}{\partial u_i} (u_i - u_i^{(0)}) \end{array} \right\}$$

$$\begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} F_1(u^{(0)}) \\ \vdots \\ F_N(u^{(0)}) \end{pmatrix} \rightarrow \underbrace{\begin{pmatrix} \frac{\partial F_1}{\partial u_1} & \frac{\partial F_1}{\partial u_N} \\ \vdots & \vdots \\ \frac{\partial F_N}{\partial u_1} & \frac{\partial F_N}{\partial u_N} \end{pmatrix}}_{J} \begin{pmatrix} u_1 - u_1^{(0)} \\ \vdots \\ u_N - u_N^{(0)} \end{pmatrix}$$

$$\begin{pmatrix} u_1^{(1)} \\ \vdots \\ u_N^{(1)} \end{pmatrix} = \begin{pmatrix} u_1^{(0)} \\ \vdots \\ u_N^{(0)} \end{pmatrix} - J^{-1} \begin{pmatrix} F_1(u^{(0)}) \\ \vdots \\ F_N(u^{(0)}) \end{pmatrix}$$



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