# Finite Volume in 2D

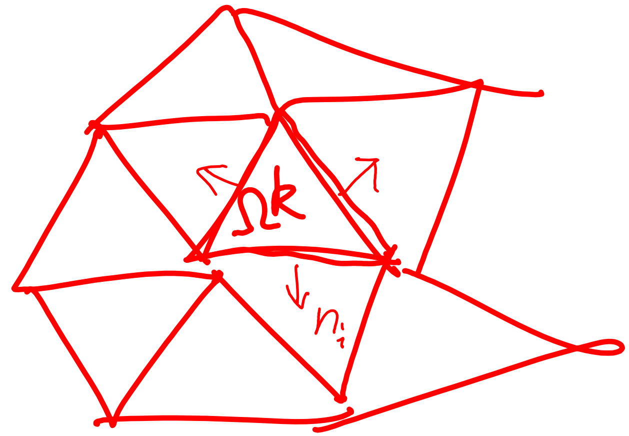
$$\frac{d}{dt}\int_{\Omega} \rho \, dx = -\int_{\partial\Omega} \vec{n} \cdot \vec{F}(\rho) \, ds$$

$$\frac{d}{dt}\int_{L}^{R} \rho \, dx = \vec{F}(\rho(L)) - \vec{F}(\rho(R))$$

$$\bar{\rho}_k = \frac{\int_L^R \rho \, dx}{\Delta x_k}$$

$$\frac{d}{dt}\bar{\rho}_k = \frac{1}{\Delta x_k}\left(\vec{F}_{k-\frac{1}{2}} - \vec{F}_{k+\frac{1}{2}}\right)$$

$$\bar{\rho}_k = \frac{\int_{\Omega_k} \rho \, dx}{A_k}$$




$$\frac{d\bar{\rho}_k}{dt} = \frac{1}{A_k}\frac{d}{dt}\int_{\Omega_k} \rho \, dx$$

$$= \frac{-1}{A_k} \cdot \int_{\partial\Omega_k} \vec{n} \cdot F \, ds$$

$$= -\frac{1}{A_k} \sum_{i \in S_k} \vec{n}_i \cdot \vec{F}_i \, S_i$$

$$F(\bar{\rho}_k, \bar{\rho}_{neighbor})$$

turn PDE $\rightarrow$ ODE

$$\frac{d}{dt}\begin{pmatrix} \bar{\rho}_1 \\ \bar{\rho}_2 \\ \vdots \\ \bar{\rho}_N \end{pmatrix} = F\begin{pmatrix} \bar{\rho}_1 \\ \vdots \\ \bar{\rho}_N \end{pmatrix}$$

# Order of accuracy

$$\frac{du}{dt} = f(u)$$

$$\frac{\delta}{\delta t} u - f(u) \neq 0$$

FE

$$\frac{u^{k+1} - u^k}{\Delta t} - f(u^k) \neq 0$$

$$= O(\Delta t^p)$$

$p$ is local order

$$\tau = u^{k+1} - \left(u^k + \Delta t\, f(u^k)\right) = O(\Delta t^{p+1})$$

Local

Global $=$ Local

iff zero stable

under that $p \geq 1$ local.

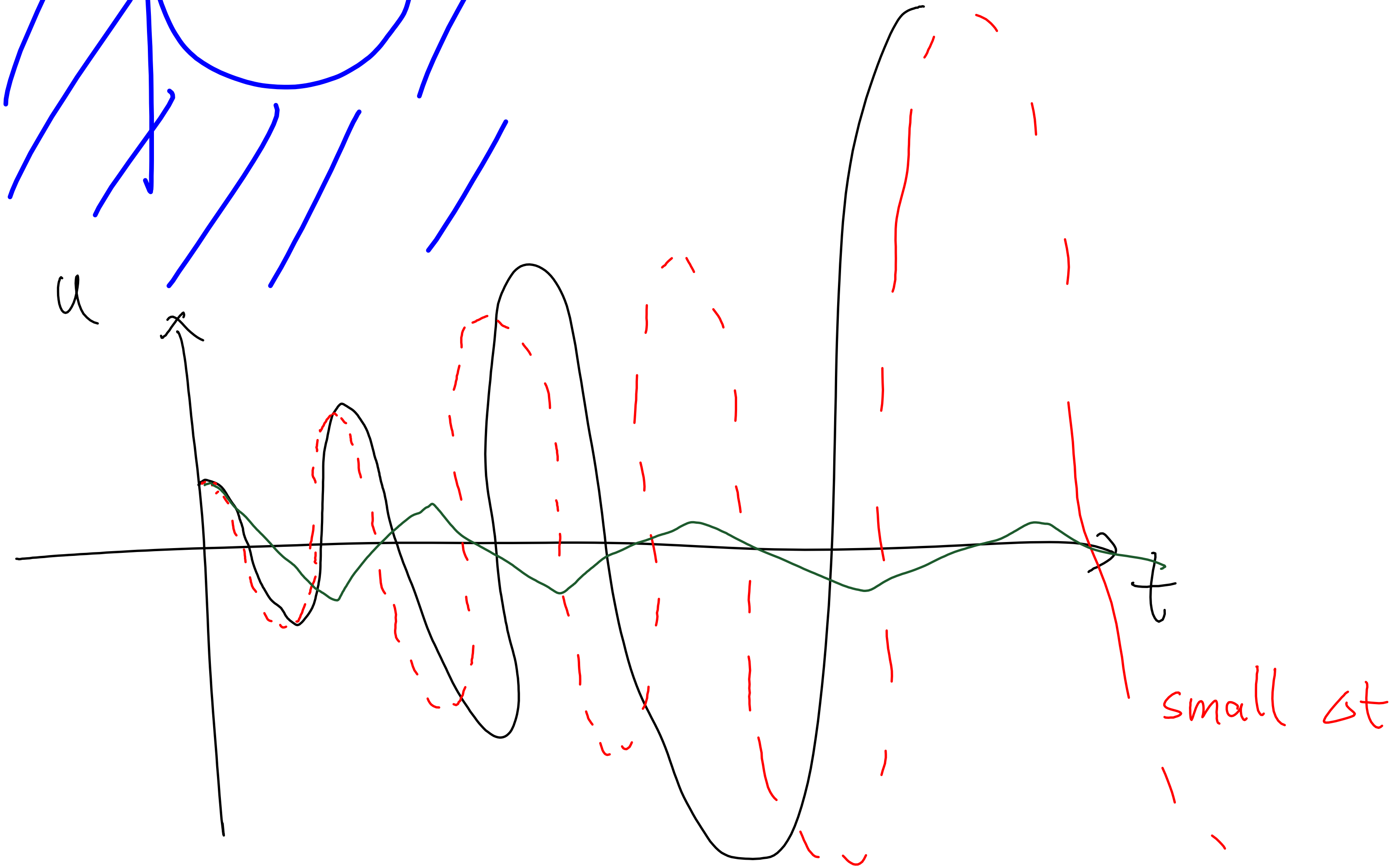
# Eigenvalue stability

$v^k$ is bounded for $\frac{du}{dt} = \lambda u$ eigen

$\frac{du}{dt} = 0$ zero

scheme $(\lambda \Delta t)$


1
2


u
t
small Δt

# Stiffness and Newton-Raphson

Implicit

Lots of coding

Solve nonlinear Eqn

Larger stability region
in stiff problems

Explicit

$$F(u) = 0 \qquad \frac{u - u^k}{\Delta t} = f(u, u^k, \cdots)$$

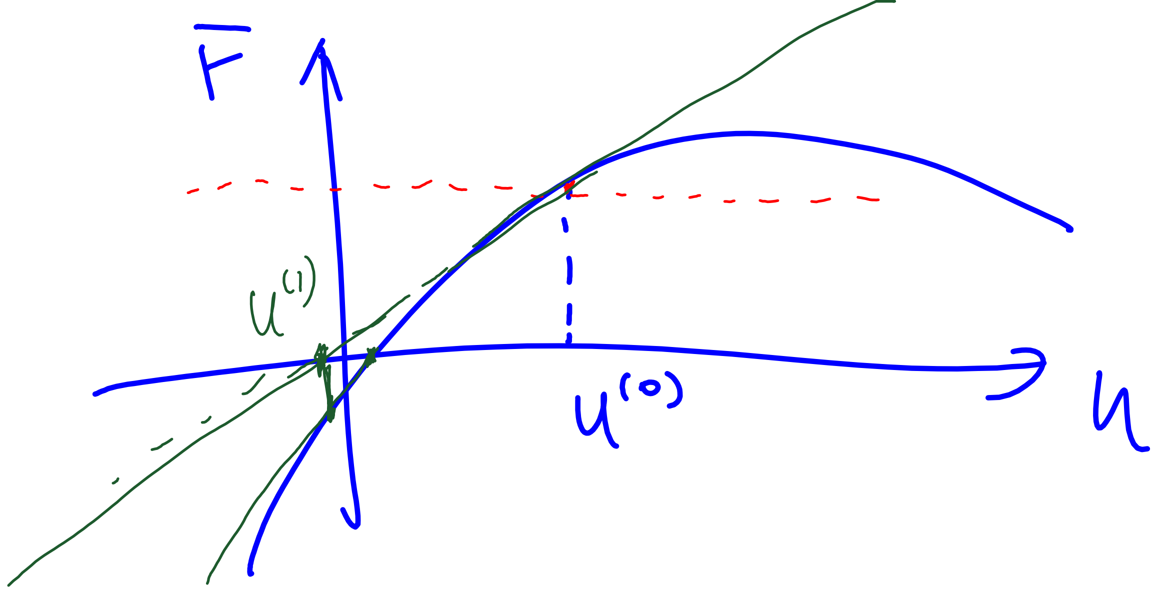
$$F: \frac{u - u^k}{\Delta t} - f(u, u^k, \cdots)$$

$$u^{(0)} = u^k$$

$$\begin{cases} F_1(u) \approx F_1(u^{(0)}) + \sum_{i=1}^{N} \frac{\partial F_1}{\partial u_i}(u_i - u_i^{(0)}) \\ \vdots \\ F_N(u) \approx F_N(u^{(0)}) + \sum_{i=1}^{N} \frac{\partial F_N}{\partial u_i}(u_i - u_i^{(0)}) \end{cases}$$

$$\begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} F_1(u^{(0)}) \\ \vdots \\ F_N(u^{(0)}) \end{pmatrix} + \underbrace{\begin{pmatrix} \frac{\partial F_1}{\partial u_1} & \cdots & \frac{\partial F_1}{\partial u_N} \\ \vdots & & \\ \frac{\partial F_N}{\partial u_1} & \cdots & \frac{\partial F_N}{\partial u_N} \end{pmatrix}}_{J} \begin{pmatrix} u_1 - u_1^{(0)} \\ \vdots \\ u_N - u_N^{(0)} \end{pmatrix}$$

$$\begin{pmatrix} u_1^{(1)} \\ \\ u_N^{(1)} \end{pmatrix} = \begin{pmatrix} u_1^{(0)} \\ \vdots \\ u_N^{(0)} \end{pmatrix} - J^{-1} \begin{pmatrix} F_1(u^{(0)}) \\ \vdots \\ F_N(u^{(0)}) \end{pmatrix}$$


F
$u^{(1)}$
$u^{(0)}$
u