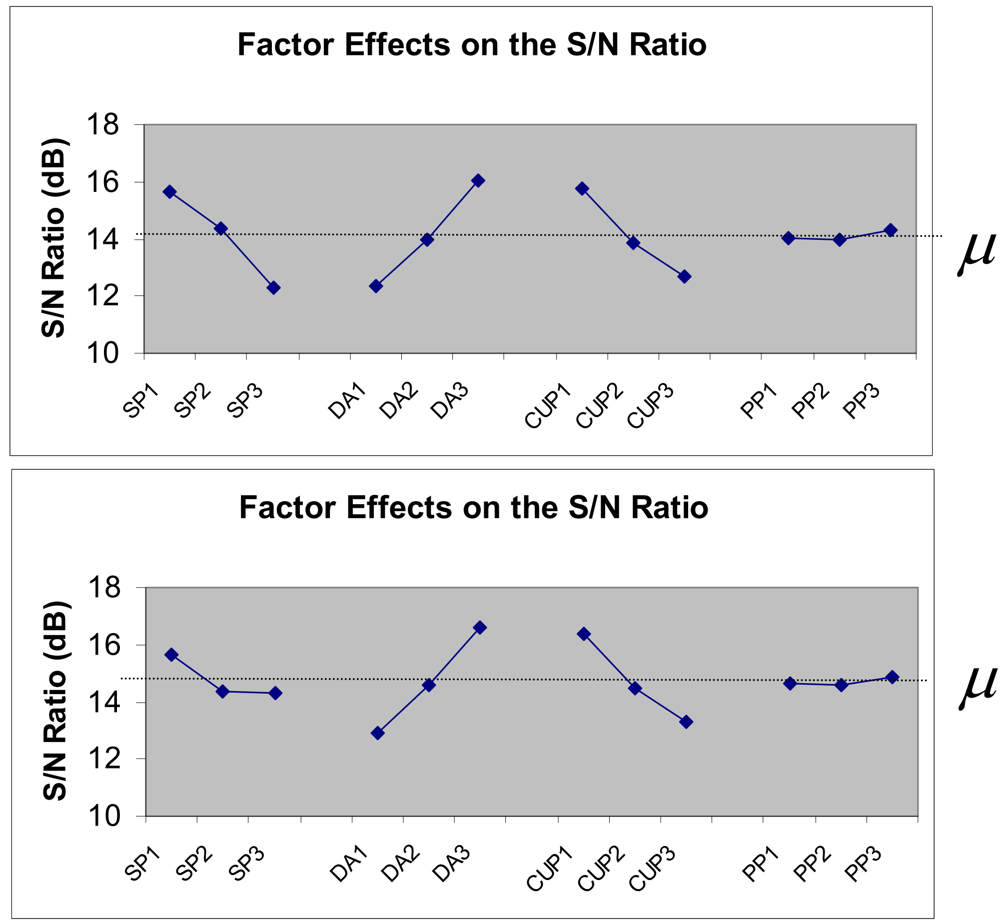
# Plan for the Session

- Questions?
- Complete some random topics
- Lecture on Design of Dynamic Systems (Signal / Response Systems)
- Recitation on HW#5?

# Dummy Levels and $\mu$

- Before
- After
  - Set SP2=SP3
  - $\mu$ rises
  - Predictions unaffected



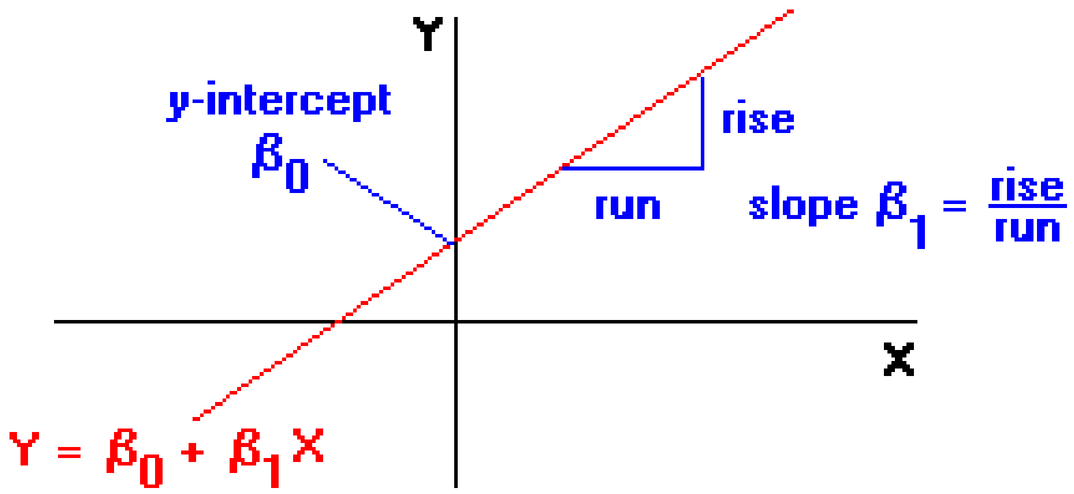
# Number of Tests

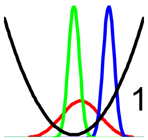
- One at a time
  - Listed as small
- Orthogonal Array
  - Listed as small
- White Box
  - Listed as medium

# Linear Regression

- Fits a linear model to data

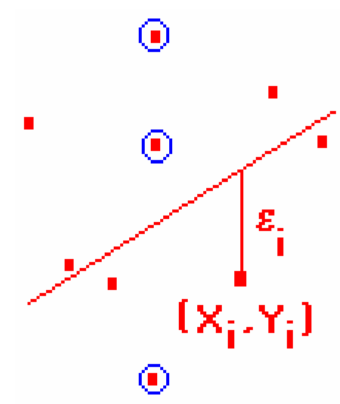
$$Y_i = \beta_0 + \beta_1 \cdot X_i + \varepsilon_i$$

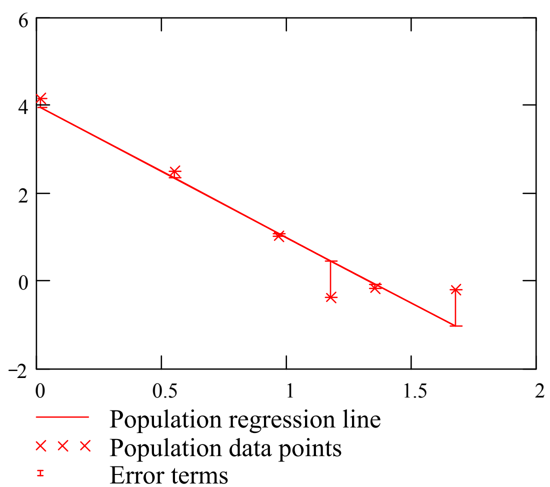


# Error Terms

- Error should be independent
  - Within replicates
  - Between X values



# Least Squares Estimators

- We want to choose values of $b_o$ and $b_1$ that minimize the sum squared error

$$SSE(b_0, b_1) := \sum_i \left[ \mathbf{y}_i - (b_0 + b_1 \cdot \mathbf{x}_i) \right]^2$$

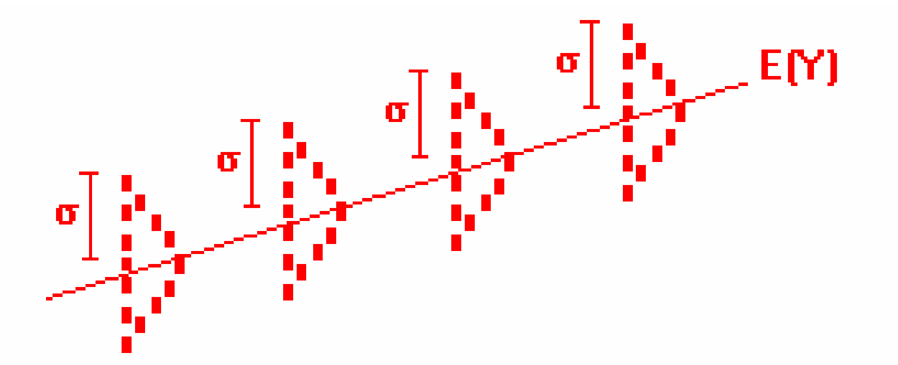
- Take the derivatives, set them equal to zero and you get

$$b_1 := \frac{\sum_i (\mathbf{x}_i - \mathrm{mean}(\mathbf{x})) \cdot (\mathbf{y}_i - \mathrm{mean}(\mathbf{y}))}{\sum_i (\mathbf{x}_i - \mathrm{mean}(\mathbf{x}))^2}$$

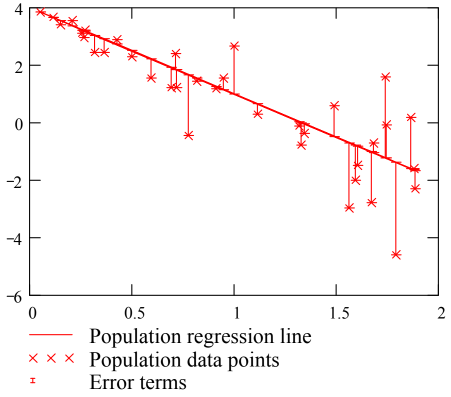
$$b_0 := \mathrm{mean}(\mathbf{y}) - b_1 \cdot \mathrm{mean}(\mathbf{x})$$

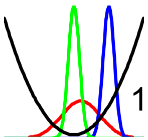
# Distribution of Error

- Homoscedasticity



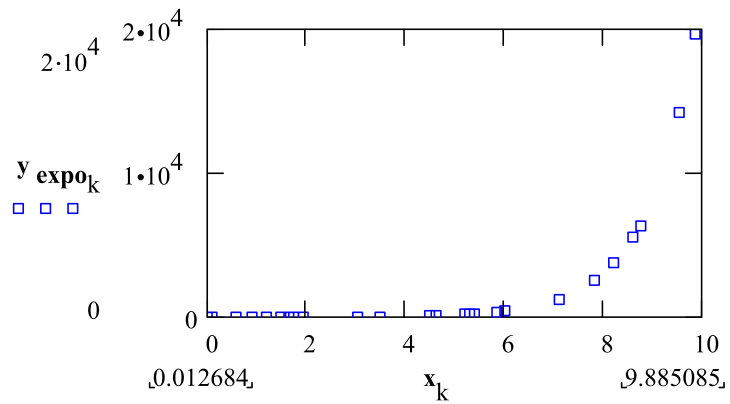

- Heteroscedasticity

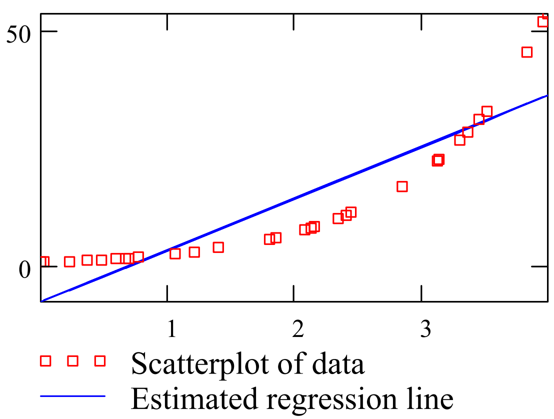


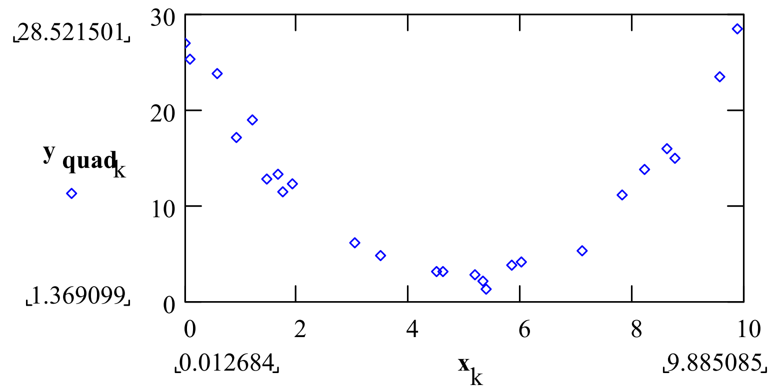



# Cautions Re: Regression

- What will result if you run a linear regression on these data sets?



# Linear Regression Assumptions

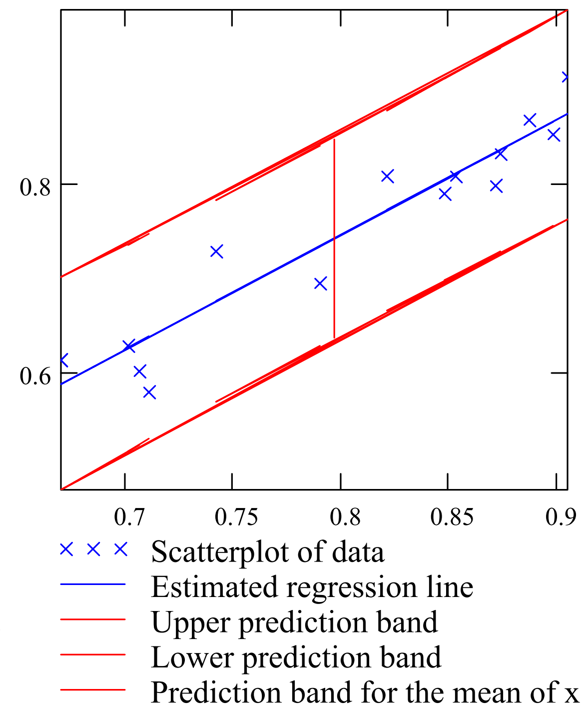
1. The average value of the dependent variable $Y$ is a linear function of $X$.

2. The only random component of the linear model is the error term $\varepsilon$. The values of $X$ are assumed to be fixed.

3. The errors between observations are uncorrelated. In addition, for any given value of $X$, the errors are are normally distributed with a mean of zero and a constant variance.

# If The Assumptions Hold

- You can compute confidence intervals on $\beta_1$
- You can test hypotheses
  - Test for zero slope $\beta_1=0$
  - Test for zero intercept $\beta_0=0$
- You can compute prediction intervals

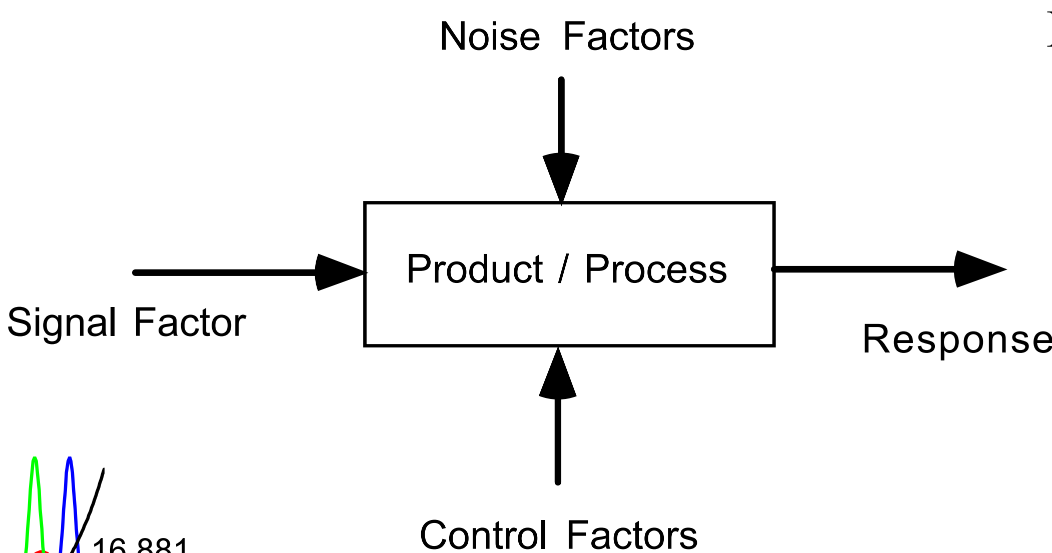


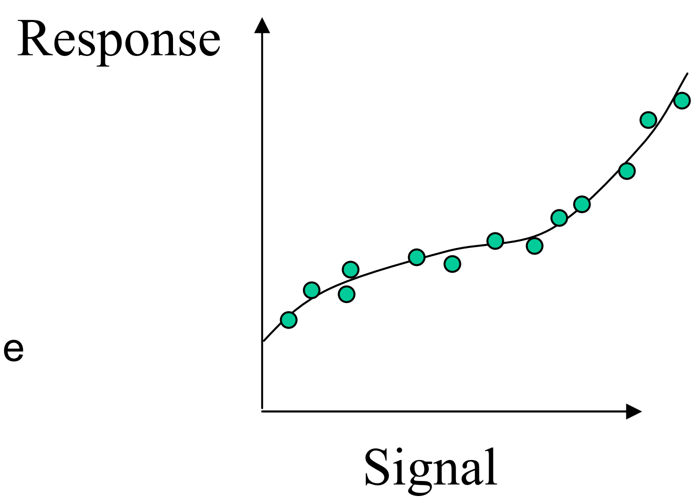
# Design of Dynamic Systems (Signal / Response Systems)

# Dynamic Systems Defined

"Those systems in which we want the system response to follow the levels of the signal factor in a prescribed manner"

– Phadke, pg. 213



# Examples of Dynamic Systems

- Calipers
- Automobile steering system
- Aircraft engine
- Printing
- Others?

# Static versus Dynamic

**Static**
- Vary CF settings
- For each row, induce noise
- Compute S/N for each row (single sums)

**Dynamic**
- Vary CF settings
- Vary signal (M)
- Induce noise
- Compute S/N for each row (double sums)

# S/N Ratios for Dynamic Problems

| | | Signals | |
|---|---|---|---|
| | | Continuous | Digital |
| Responses | Continuous | C-C | C-D |
| | Digital | D-C | D-D |

Examples of each?

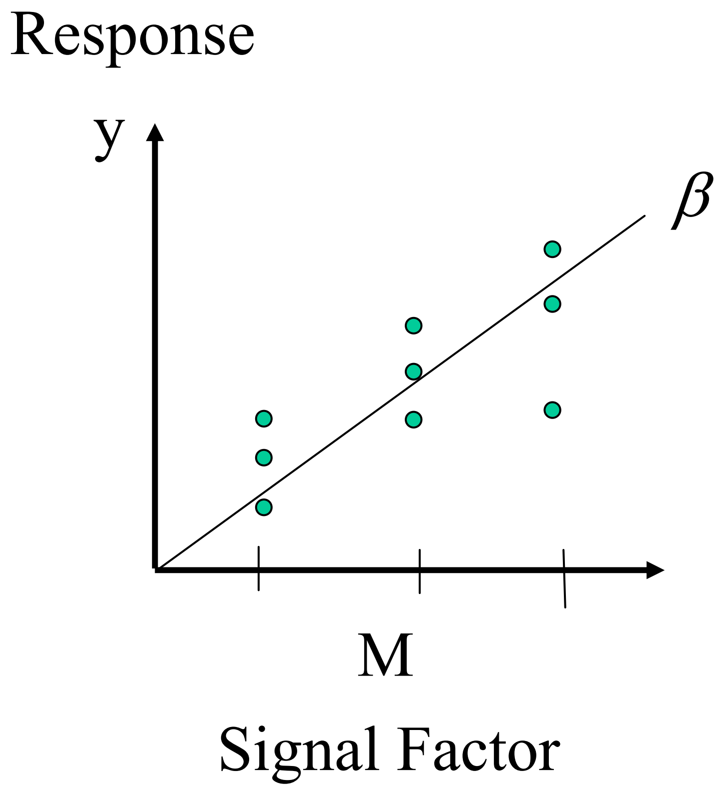
# Continuous - continuous S/N

- Vary the signal among discrete levels
- Induce noise, then compute

$$\beta = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} y_{ij} M_i}{\sum_{i=1}^{m}\sum_{j=1}^{n} {M_i}^2}$$

$$\sigma_e{}^2 = \frac{1}{mn-1}\sum_{i=1}^{m}\sum_{j=1}^{n}(y_{ij} - \beta M_i)^2$$

$$\eta = 10\log_{10}\frac{\beta^2}{\sigma_e{}^2}$$



# C - C S/N and Regression

## C-C S/N

$$\beta = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} y_{ij} M_i}{\sum_{i=1}^{m}\sum_{j=1}^{n} M_i^{\ 2}}$$

$$\sigma_e^{\ 2} = \frac{1}{mn-1}\sum_{i=1}^{m}\sum_{j=1}^{n}(y_{ij} - \beta M_i)^2$$

$$\eta = 10\log_{10}\frac{\beta^2}{\sigma_e^{\ 2}}$$

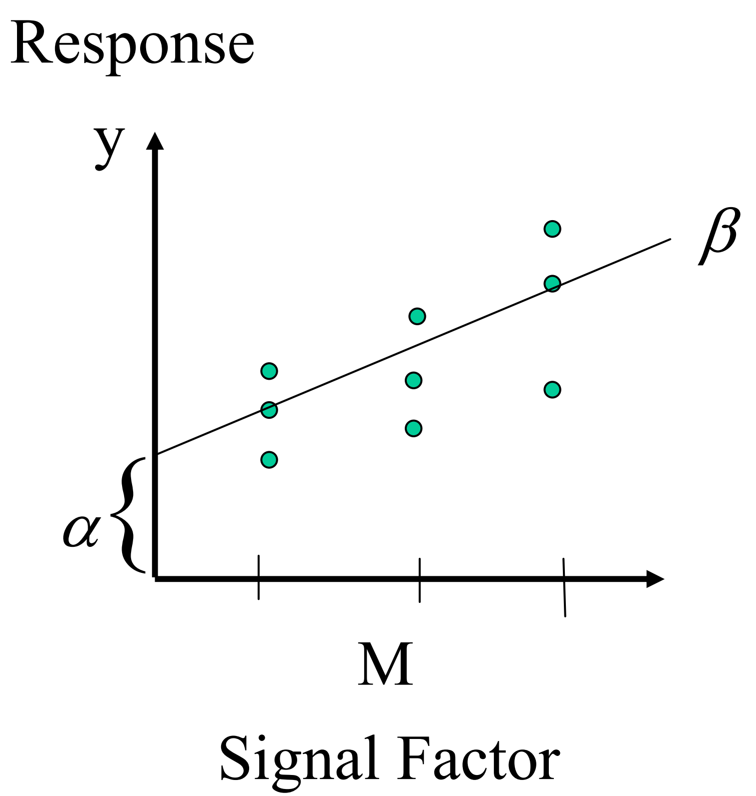
## Linear Regression

$$b_1 := \frac{\sum_i \left(\mathbf{x}_i - \mathrm{mean}(\mathbf{x})\right)\cdot\left(\mathbf{y}_i - \mathrm{mean}(\mathbf{y})\right)}{\sum_i \left(\mathbf{x}_i - \mathrm{mean}(\mathbf{x})\right)^2}$$

$$\mathrm{SSE}\left(b_0, b_1\right) := \sum_i \left[\mathbf{y}_i - \left(b_0 + b_1 \cdot \mathbf{x}_i\right)\right]^2$$

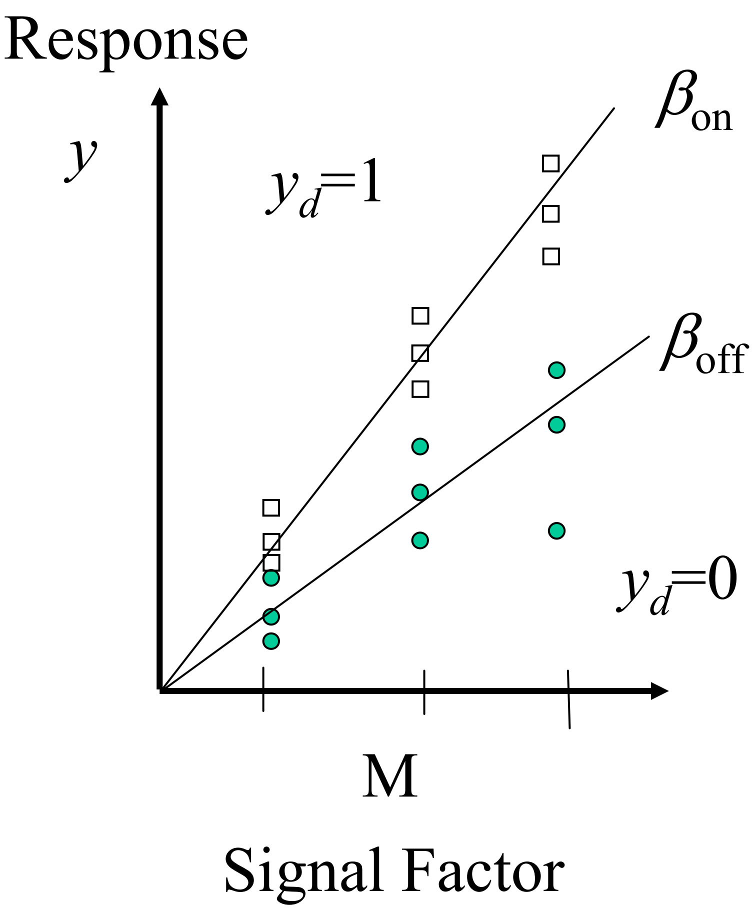
# Non-zero Intercepts

- Use the same formula for S/N as for the zero intercept case
- Find a second scaling factor to independently adjust $\beta$ and $\alpha$



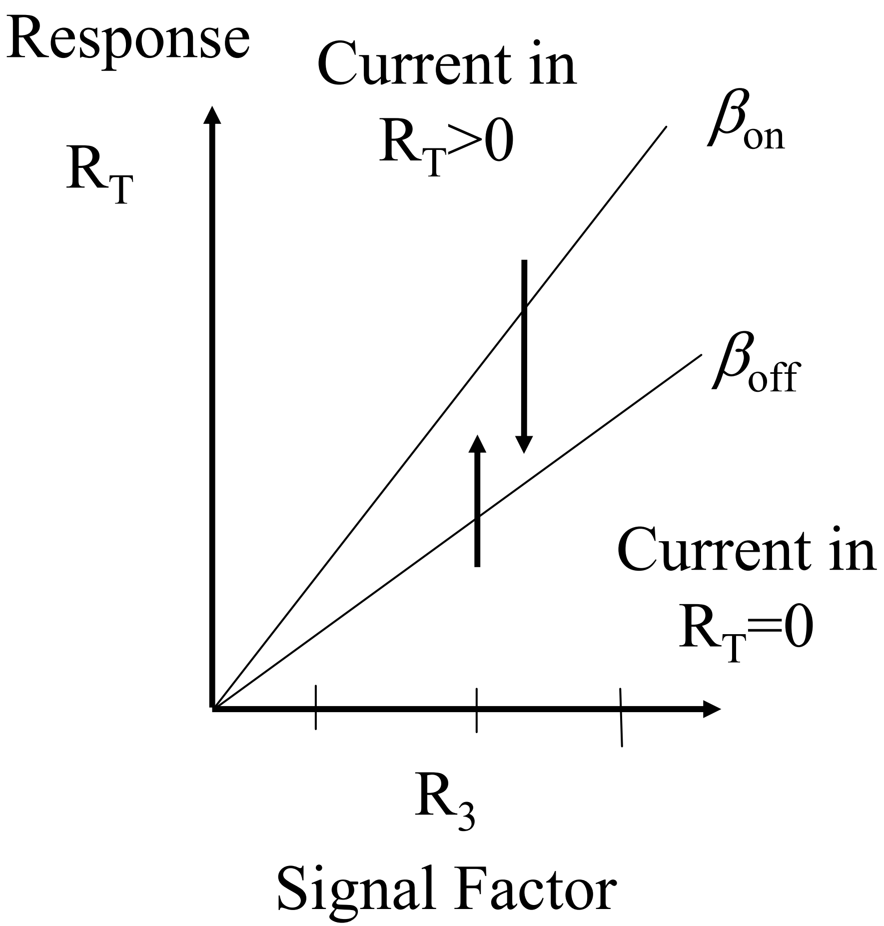
# Continuous - digital S/N

- Define some continuous response $y$
- The discrete output $y_d$ is a function of $y$



# Temperature Control Circuit

- Resistance of thermistor decreases with increasing temperature
- Hysteresis in the circuit lengthens life



# System Model

- Known in closed form

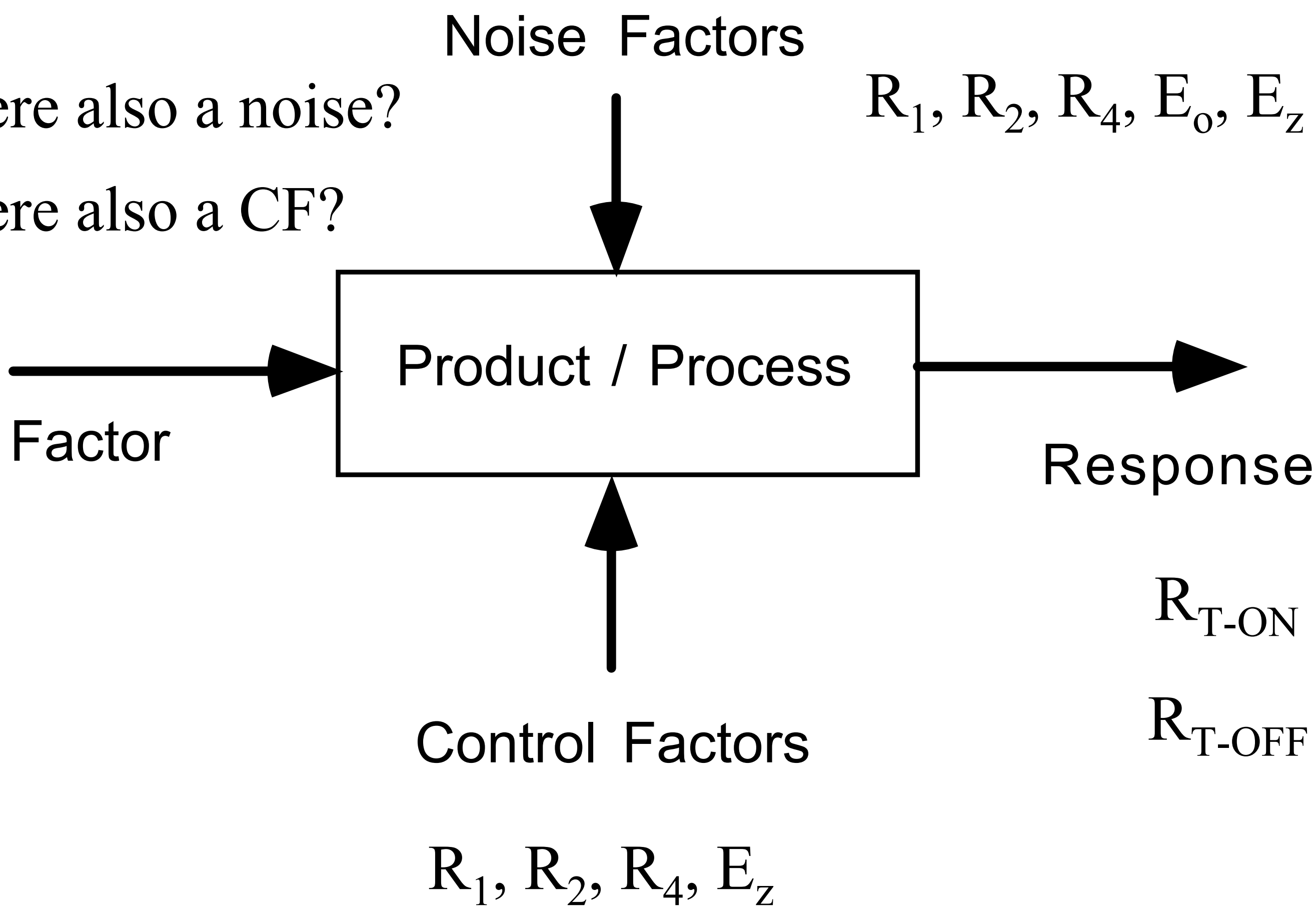
$$R_{T\_ON} = \frac{R_3 \cdot R_2 \cdot \left(E_z \cdot R_4 + E_o \cdot R_1\right)}{R_1 \cdot \left(E_z \cdot R_2 + E_z \cdot R_4 - E_o \cdot R_2\right)}$$

$$R_{T\_OFF} = \frac{R_3 \cdot R_2 \cdot R_4}{R_1 \cdot \left(R_2 + R_4\right)}$$

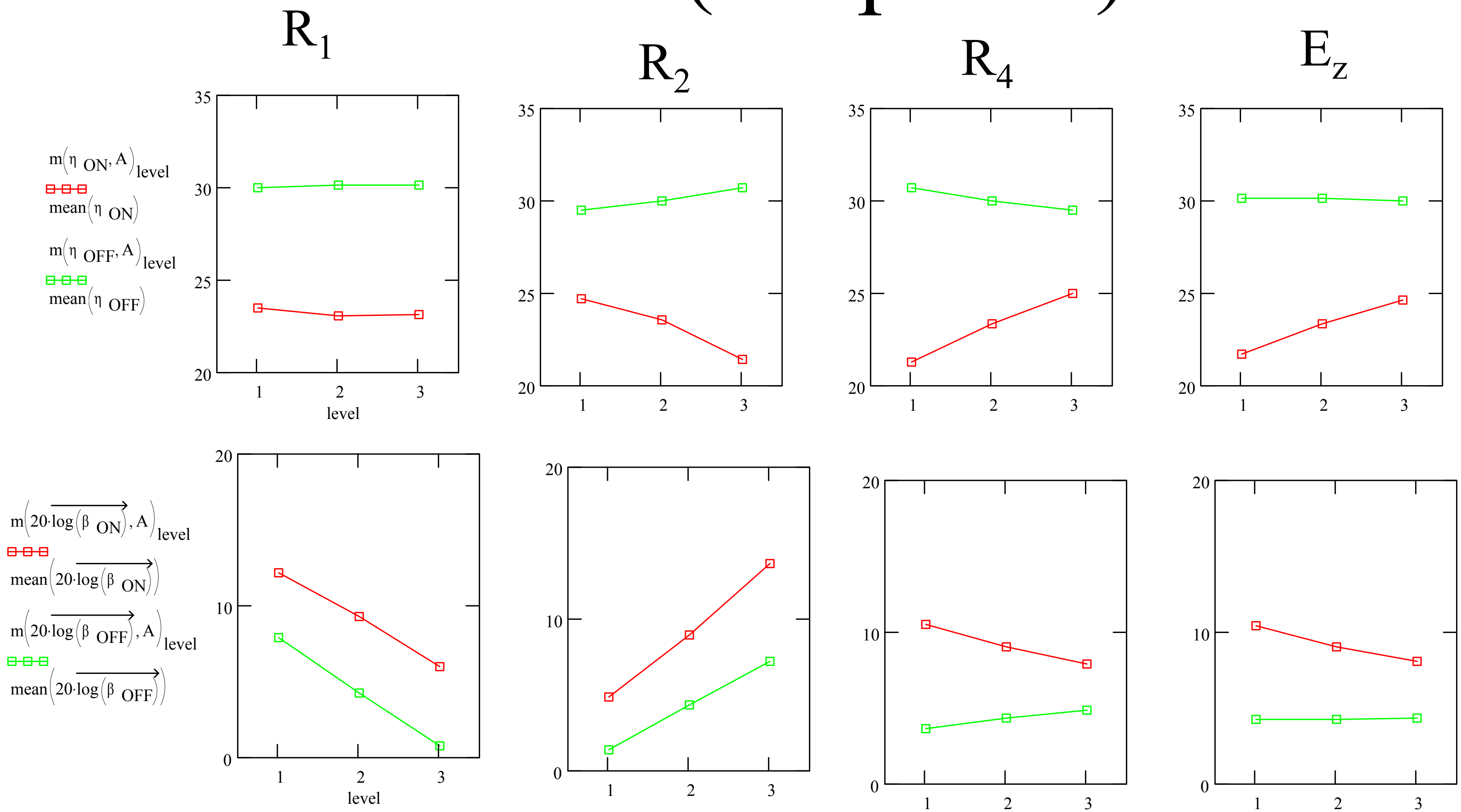
# Problem Definition

What if $R_3$ were also a noise?

What if $R_3$ were also a CF?



# Results (Graphical)



# Results (Interpreted)

- $R_T$ has little effect on either S/N ratio
  - Scaling factor for <u>both</u> $\beta$s
  - What if I needed to independently set $\beta$s?
- Effects of CFs on $R_{T\text{-}OFF}$ smaller than for $R_{T\text{-}ON}$
- Best choices for $R_{T\text{-}ON}$ tend to negatively impact $R_{T\text{-}OFF}$
- Why not consider factor levels outside the chosen range?

# Next Steps

- Homework #8 due 7 July
- Next session Monday 6 July 4:10-6:00
  - Read Phadke Ch. 9 -- "Design of Dynamic Systems"
  - No quiz tomorrow
- 6 July -- Quiz on Dynamic Systems