Trajectory Design For A Visible Geosynchronous Earth Imager

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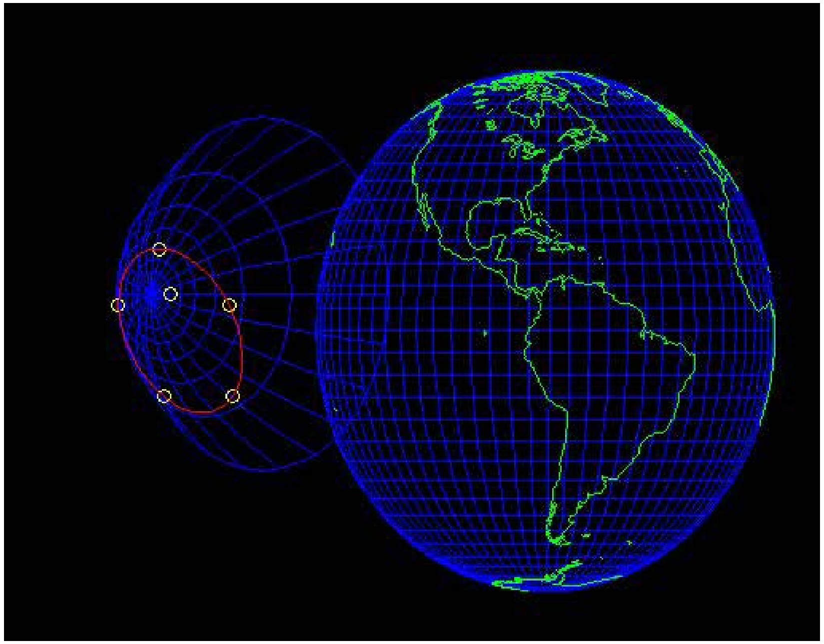
Introduction

Objective : ***To compare the different imaging configurations for a Separated Spacecraft Interferometer operating from an Earth's orbit***

Outline :

- Interferometric requirements & Orbit Selection
- Equations of Motions (Hill's Equations)
- Steered Planar Array
- Propellant Free Array: Collector S/C
- Results
- Summary

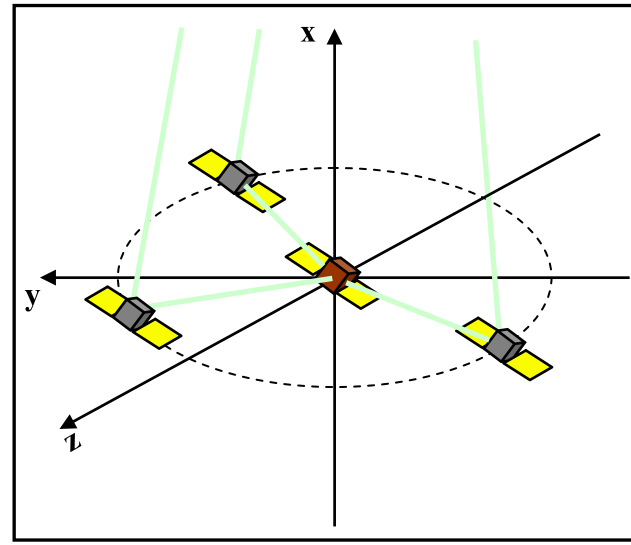


Interferometric Requirements & Orbit Selection

Interferometric Requirements:

Reqt 1. Equal science light pathlength for visible imaging

Reqt 2. Axi-symmetric angular resolution about LOS



Far-field assumption

- Array sees planar wavefronts from targets

Orbit Selection: Geosynchronous

- Higher altitude, lower perturbative effects (eg. J_2)

Equations of Motions

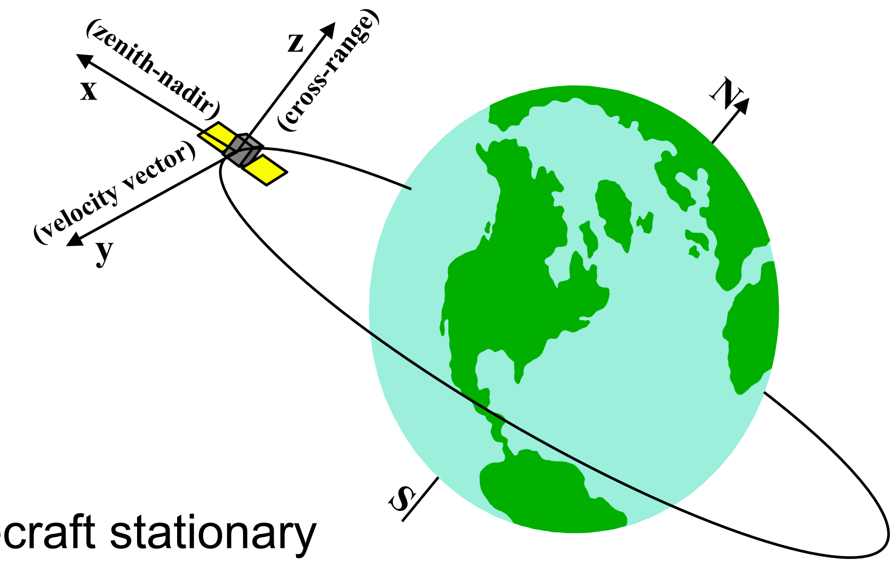
Assumption : First order perturbation about natural circular Keplerian orbit

Hill's Equations :

$$a_x = \ddot{x} - 3n^2 x - 2n\dot{y}$$

$$a_y = \ddot{y} + 2n\dot{x}$$

$$a_z = \ddot{z} + n^2 z$$



Total Spacecraft Velocity Increment :

$$\Delta V = \int_0^{T_{life}} \sqrt{a_x^2 + a_y^2 + a_z^2}\, dt$$

Example : ΔV required to hold a spacecraft stationary at (x,y,z)

Spacecraft instantaneous acceleration :

$$a_x = -3n^2 x \qquad a_y = 0 \qquad a_z = n^2 z$$

ΔV required :

$$\Delta V = n^2 T_{life} \sqrt{9x^2 + z^2}$$

DSS Architecture 1

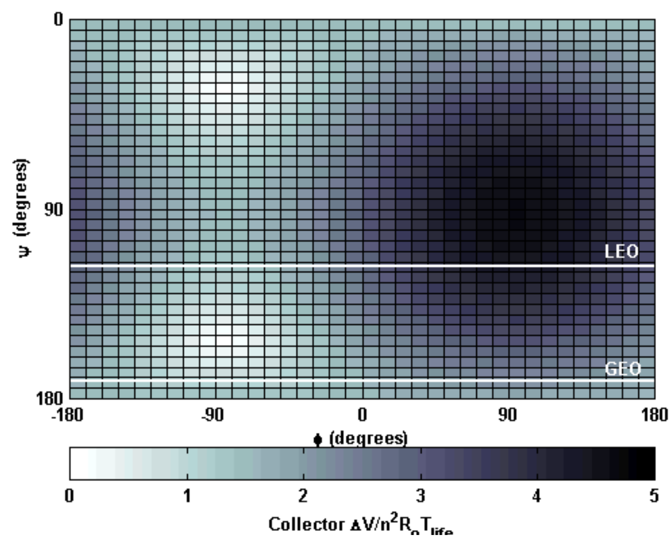
Constraint collector spacecraft to a local horizontal circular trajectory with combiner spacecraft at the center (Reqts. 1 & 2)

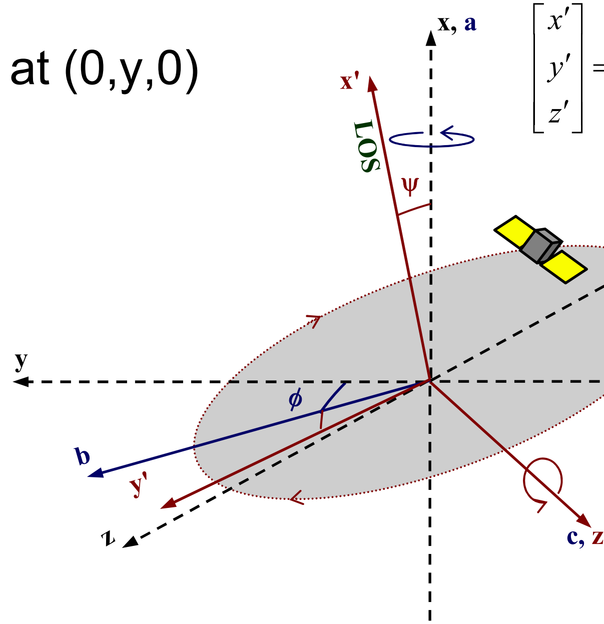
ΔV Requirement

- No ΔV for stationary combiner spacecraft at (0,y,0)
- ΔV for collector spacecraft

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{Hill} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \sin\phi \end{bmatrix} \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 0 \\ \pm R_o \sin(nt+\alpha) \\ R_o \cos(nt+\alpha) \end{bmatrix}$$





Average collector s/c ΔV at GEO altitude :

$$\Delta V / n^2 R_o T_{life} = 1.55$$

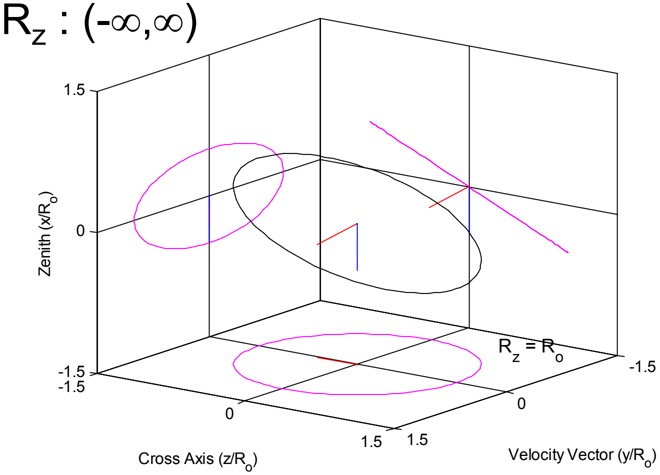
DSS Architecture 2

Constraint the projection of the collector spacecraft's trajectory to circular (Reqt. 2)

- Propellant free trajectories - (Project 2 x 1 ellipse in velocity plane)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{Collector} = \begin{bmatrix} \pm (R_o/2)\cos nt \\ \mp R_o \sin nt \\ R_z \cos nt \end{bmatrix}$$

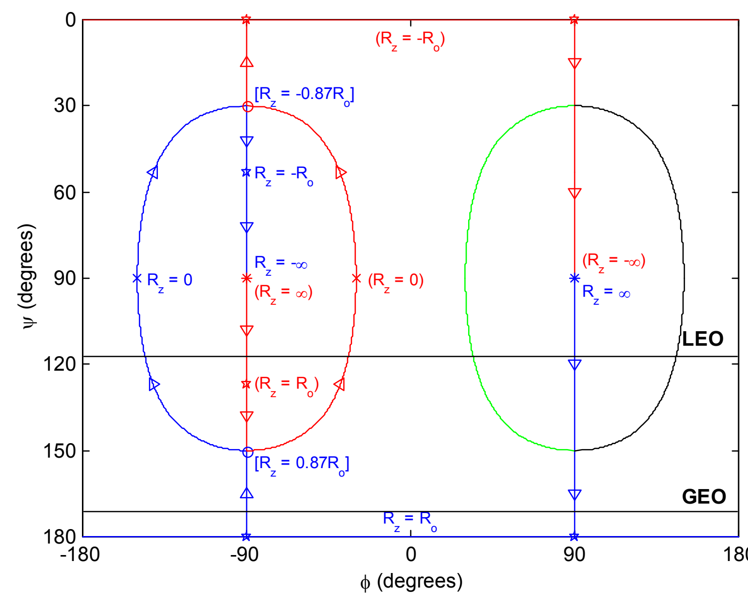
Vary R_z : $(-\infty,\infty)$



Intersection between a plane and a circular paraboloid results in an ellipse

- Placed combiner spacecraft placed at focus for equal pathlength (Reqt. 1)
- for $R_z = R_o$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{Focus} = \begin{bmatrix} \left(16R_o^2 - 3p^2\right)/\left(16p\right) \\ 0 \\ \pm p/4 \end{bmatrix}$$

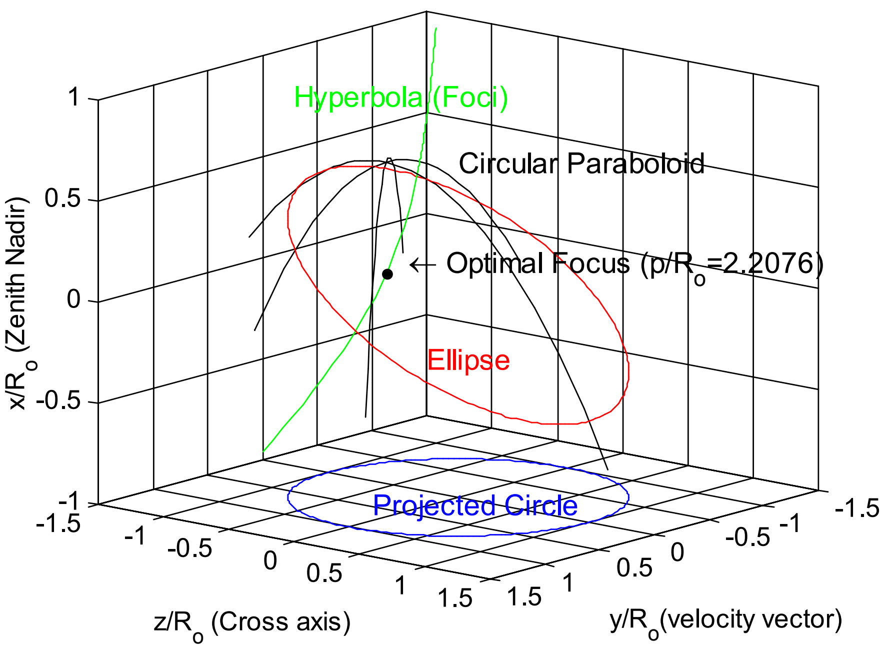


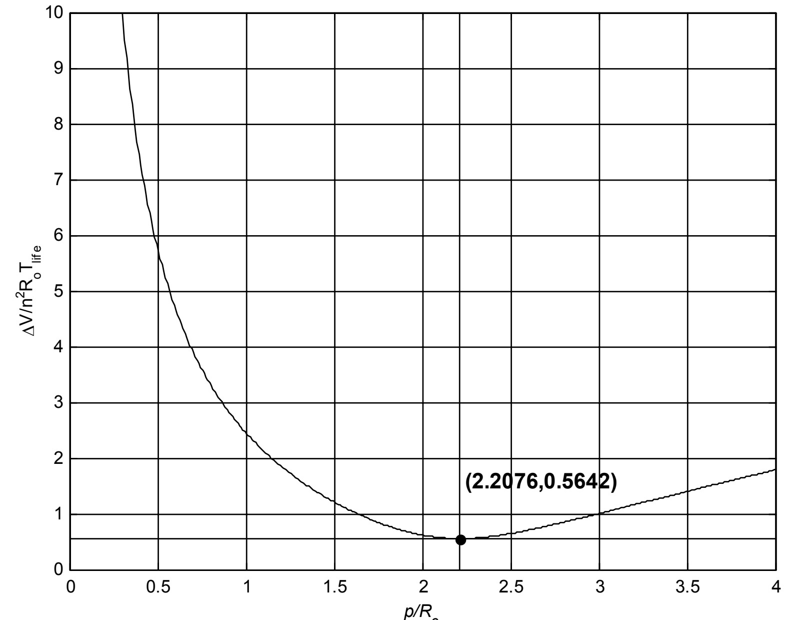
DSS Architecture 2 (cont.)

A family of paraboloids can fit onto the free elliptical trajectories

- Locus of foci maps out a hyperbola
- for $R_z = R_o$

$$x = \frac{R_o^2 - 3z^2}{\pm 4z}$$





Optimum focus :

$$p = 2.2076 R_o$$

$$\Delta V = 0.5642 n^2 R_o T_{life}$$

ΔV requirement:

- No ΔV required for collector spacecraft
- Only need ΔV to hold combiner spacecraft at paraboloid's focus

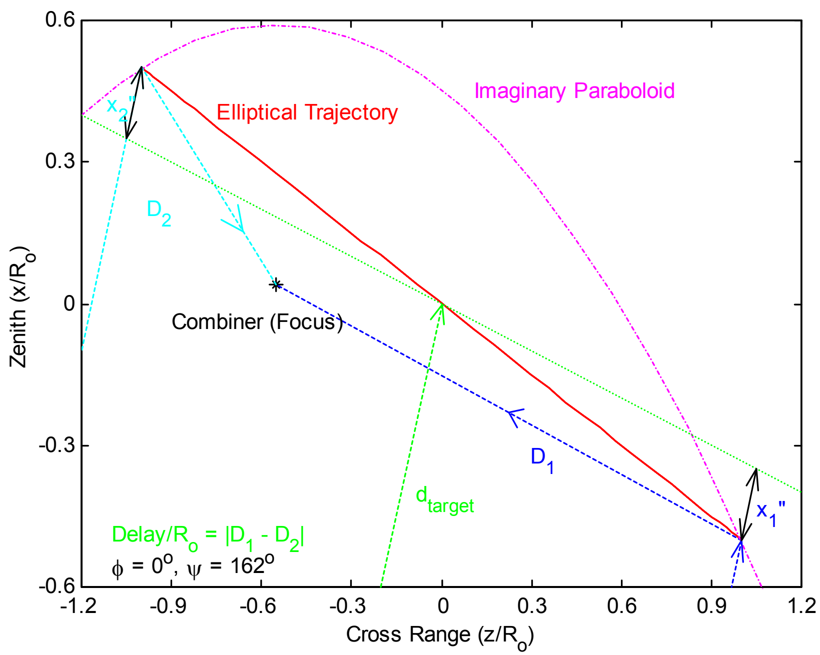
Optical Delay Lines

Steering with optical delay lines

- Collector s/c follow $R_z = R_o$ elliptical trajectory from Architecture 2
- Delay lines to image off-nadir targets (Reqt. 1)

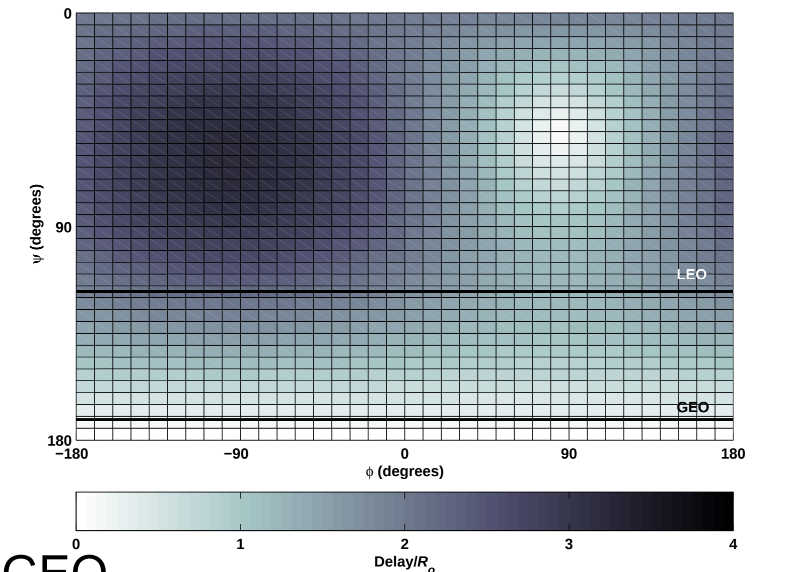
Collector-Combiner s/c distance:

$$D = R_o \sqrt{\frac{(\cos nt)^2}{4} + \left(\frac{1}{P_n} + \frac{5}{16}P_n\right)\cos nt + \frac{5}{8} + \frac{25}{256}P_n^2 + \frac{1}{P_n^2}}$$



Collector s/c trajectory in propagation vector's (x') frame:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \sin\phi \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{Hill}$$



At GEO

- Maximum delay length from GEO (x',D) = $0.310R_o$
- Minimum semi-minor axis projection (y',z') = $0.914R_o$

Mission Parameters

Components	Steered Planar	ODL
Combiner S/C	182.1 kg	182.1 kg
Combiner Propellant	-	$\Delta V/(n^2 R_o T_{life}) = 0.56$
Collector S/C	87.1 kg	87.1 kg
Collector Delay Lines	-	$0.34R_o$
Collector Propellant	$\Delta V/(n^2 R_o T_{life}) = 1.55$	-

Spacecraft Mass estimates from initial Deep Space 3 (DS3) design

- T_{life} = 5 years
- R_o = 500 m (DS3 - 1000 m baseline)

Place ODL on Collector S/C

- Ease of operation
- Lower overall dry mass and therefore, lower system mass

For each spacecraft

- Determine ΔV
- Propellant mass from Rocket equation

$$\frac{m_p}{m_d} = \exp\left(\frac{\Delta V}{I_{sp} g}\right) - 1$$

m_p - Propellant Mass (kg)
m_d - Spacecraft Dry Mass (kg)
I_{sp} - Specific impulse (sec)
g - Earth's gravity (9.81 m/sec)

Impact of ODL

General Observations

- Relatively insensitive to the number of collector s/c (> 4 collector)
- Trading between propellant and ODL mass

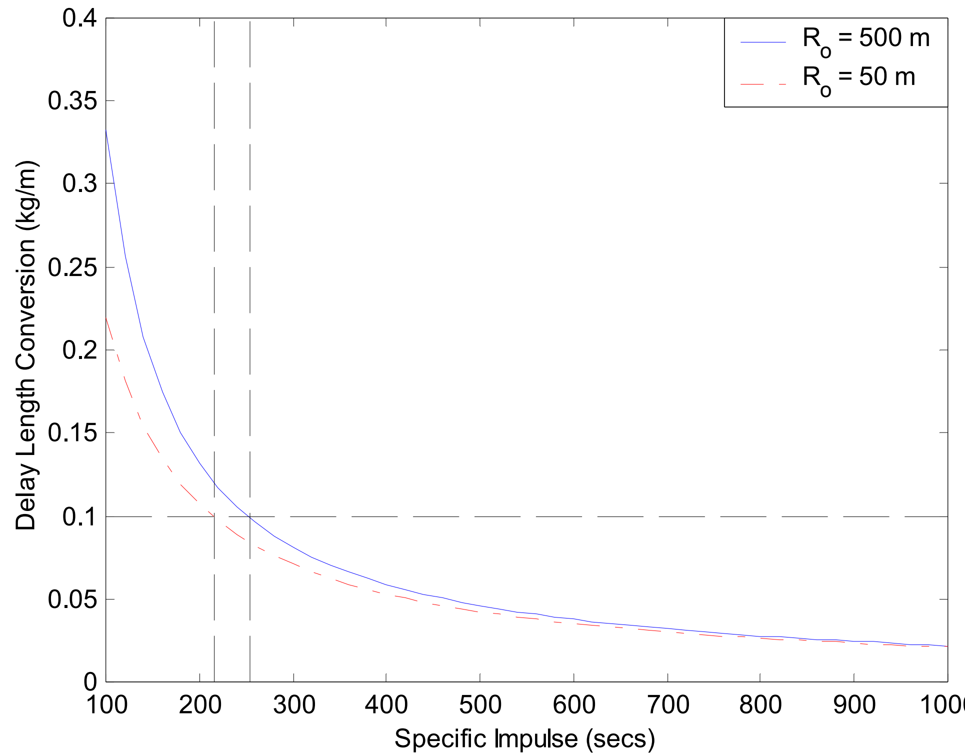
$$DLC \approx \frac{m_{coll}(\exp(\Delta V / I_{sp} g) - 1)}{0.34 R_o}$$

R_o = 500 m

- Break even point I_{sp} = 250 s (DLC = 0. 1 kg/m)
- Arch 1 : m_{comb} = 182.1, m_{coll} = 114.1
- Arch 2 : m_{comb} = 200.4, m_{coll} = 104.1

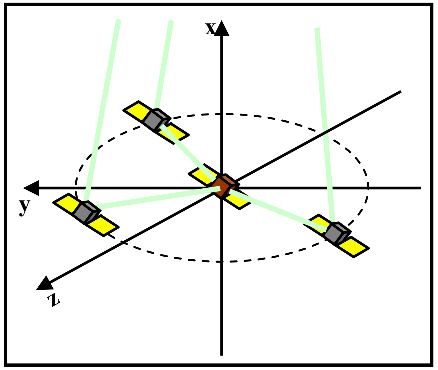
R_o = 50 m

- Break even point I_{sp} = 220 s (DLC = 0.1 kg/m)
- Arch 1 : m_{comb} = 182.1, m_{coll} = 89.7
- Arch 2 : m_{comb} = 184.1, m_{coll} = 88.8

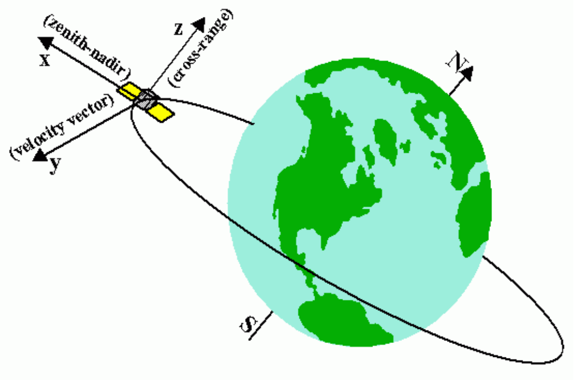


Summary (1)

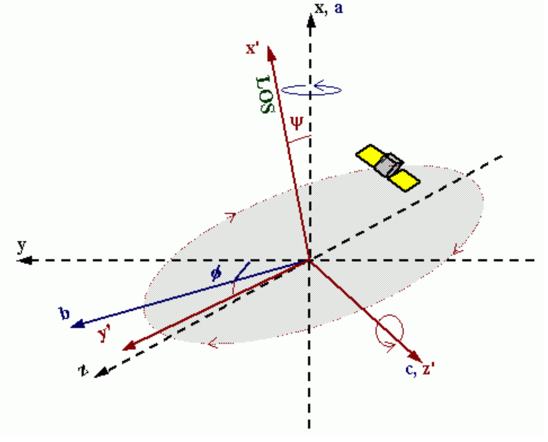
- Interferometric Requirements



- Equations of Motions
  - Hill's Equations
  - ΔV Calculation

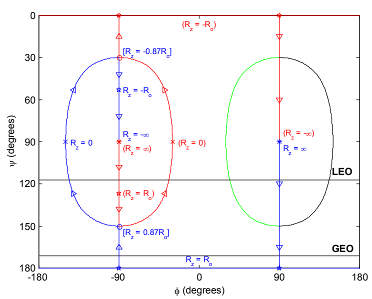


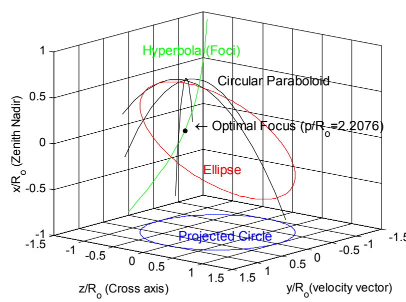
- DSS Architecture 1
  - ΔV for collector spacecraft only



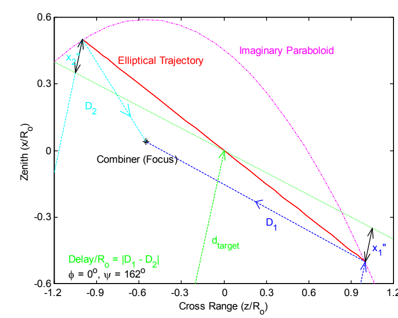
Summary (2)

- DSS Architecture 2
  - Free ΔV trajectories for collector spacecraft
  - Minimum ΔV combiner spacecraft location
  - Exploitation of conic sections





- Optical Delay Lines
  - Delay lines to steer array's LOS



- Results
  - Delay Length vs Specific Impulse cross over point