Electromagnetic Formation Flight

NRO DII Final Review

Friday, August 29, 2003

National Reconnaissance Office

Headquarters

Chantilly, VA

- Massachusetts Institute of Technology
- Space Systems Laboratory

- Lockheed Martin Corporation
- Advanced Technology Center

Outline

- **Motivation**
- **Fundamental Principles**
  - Governing Equations
  - Trajectory Mechanics
  - Stability and Control
- **Mission Applicability**
  - Sparse Arrays
  - Filled Apertures
  - Other Proximity Operations
- **Mission Analyses**
  - Sparse Arrays
  - Filled Apertures
  - Other Proximity Operations
- **MIT EMFFORCE Testbed**
  - Design
  - Calibration
  - Movie
- **Space Hardware Design Issues**
  - Thermal Control
  - Power System Design
  - High B-Field Effects
- **Conclusions**

Motivation

- Traditional propulsion uses propellant as a reaction mass

- Advantages
  - Ability to move center of mass of spacecraft
    (Momentum conserved when propellant is included)
  - Independent (and complete) control of each spacecraft

- Disadvantages
  - Propellant is a limited resource
  - Momentum conservation requires that the necessary propellant mass increase exponentially with the velocity increment (ΔV)
  - Propellant can be a contaminant to precision optics

Question I:

- Is there an alternative to using propellant?

- Single spacecraft:
  - Yes, If an external field exists to conserve momentum
  - Otherwise, not that we know of…

- Multiple spacecraft
  - Yes, again if an external field exists
  - OR, if each spacecraft produces a field that the others can react against
  - Problem: Momentum conservation prohibits control of the motion of the center of mass of the cluster, since only internal forces are present

Question II:

- Are there missions where the absolute position of the center of mass of a cluster of spacecraft does not require control?

- Yes! In fact most of the ones we can think of…
  - Image construction
    - u-v filling does not depend on absolute position
  - Earth coverage
    - As with single spacecraft, Gravity moves the mass center of the cluster as a whole, except for perturbations…
  - Disturbance (perturbation) rejection
    - The effort to control perturbations affecting absolute cluster motion (such as J2) is much greater than that for relative motion
    - Only disturbances affecting the relative positions (such as differential J2) NEED controlling to keep a cluster together
  - Docking
    - Docking is clearly a relative position enabled maneuver

Example: Image Construction

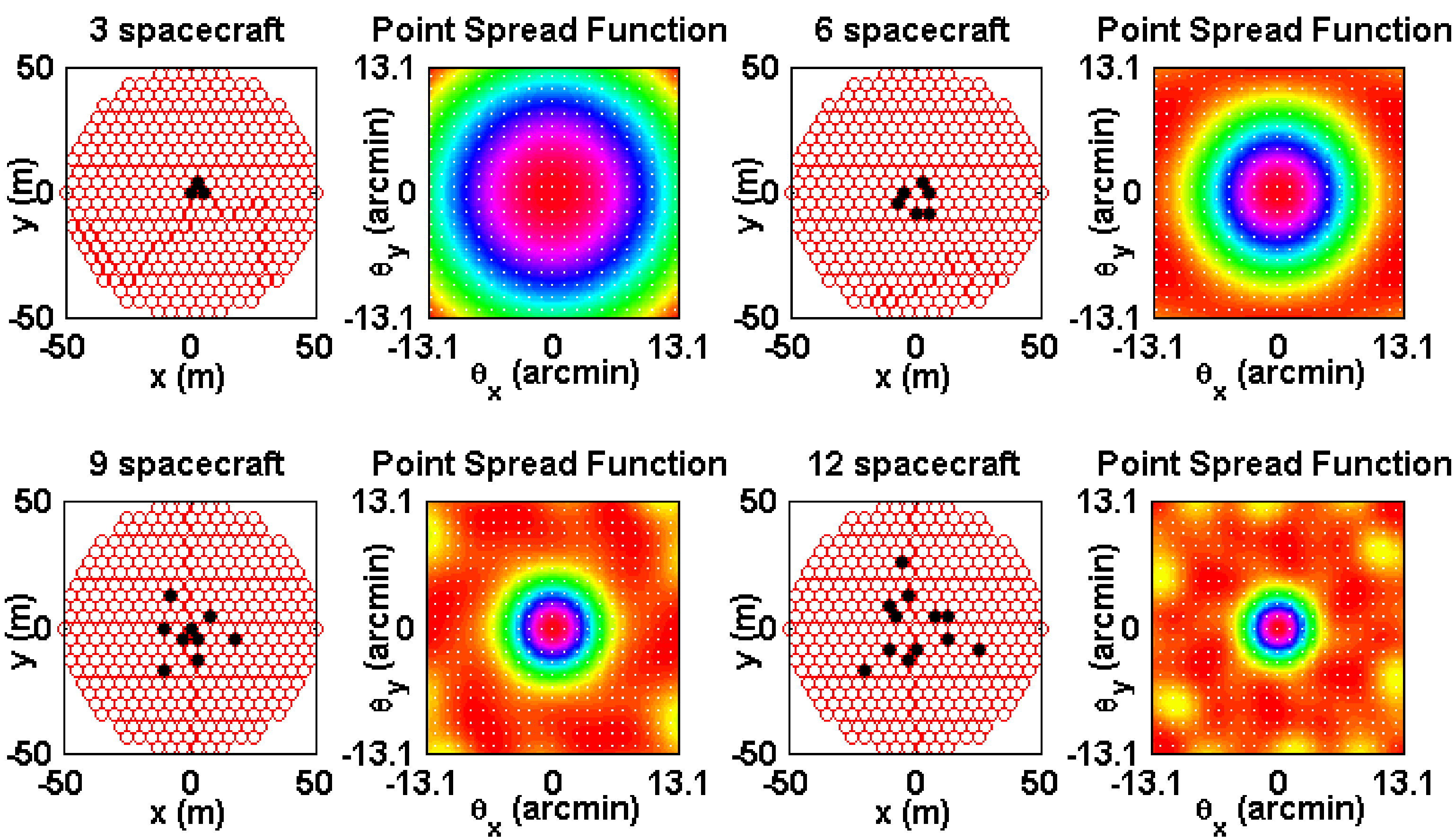
- Image quality is determined by the point spread function of aperture configuration

$$I(\psi_i,\psi_j)=\left[\underbrace{\left(\frac{\pi(1+\cos\theta)D}{\lambda}\right)\left(\frac{J_1\left(\frac{\pi D\sin\theta}{\lambda}\right)}{\frac{\pi D\sin\theta}{\lambda}}\right)}_{\text{Aperture dependence}}\underbrace{\left|\sum_{n=1}^{N}\exp\left(-\frac{2\pi i}{\lambda}(\psi_i x_n+\psi_j y_n)\right)\right|}_{\text{Geometry dependence}}\right]^2$$

- The geometry dependence can be expanded into terms which only depend on relative position

$$I(\psi_i)=I_{Ap}(\psi_i)\left[N+\cos\left(\frac{2\pi}{\lambda}\psi_i(x_1-x_2)\right)+\cos\left(\frac{2\pi}{\lambda}\psi_i(x_1-x_3)\right)+\ldots\right]$$

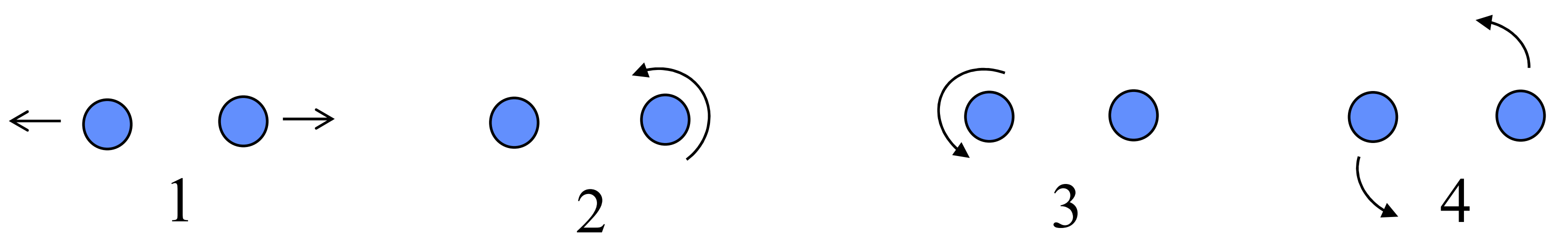
Comparison - Golay Configurations



PSFs for the Golay configurations shown here will not change if the apertures are shifted in any direction

Question III:

- What forces must be transmitted between satellites to allow for all relative degrees of freedom to be controlled?
  - In 2-D, *N* spacecraft have 3*N* DOFs, but we are only interested in controlling (and are able to control) 3*N*-2 (no translation of the center of mass)
  - For 2 spacecraft, that's a total of 4:



- All except case (4) can be generated using axial forces (such as electrostatic monopoles) and torques provided by reaction wheels
- Complete instantaneous control requires a transverse force, which can be provided using either <u>electrostatic</u> or <u>electromagnetic</u> dipoles

What is it NOT good for?

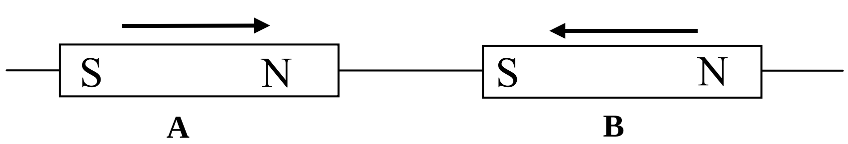
- Orbit Raising
- Bulk Plane Changes
- De-Orbit

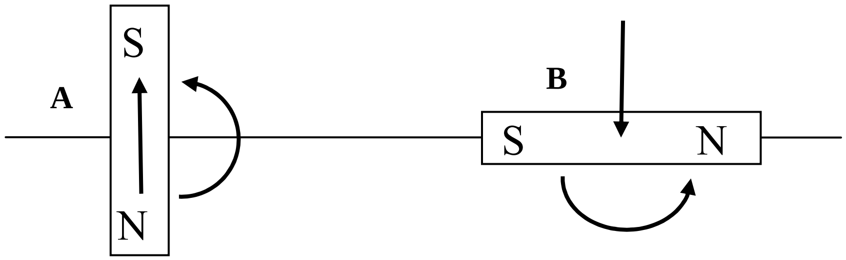
- All these require rotating the system angular momentum vector or changing the energy of the orbit

- None of these is possible using only internal forces

Forces and Torques: Conceptual

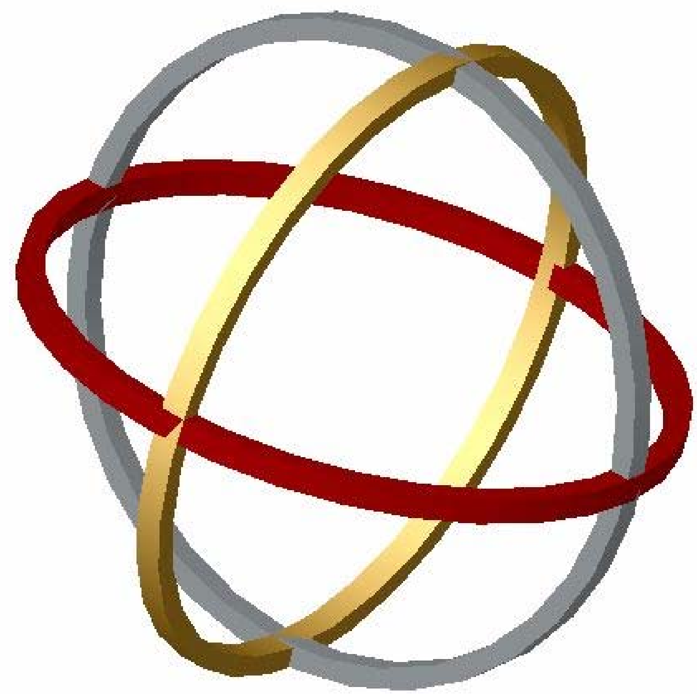
- In the Far Field, the dipole field structure for electrostatic and electromagnetic dipoles are the same
- The electrostatic analogy is useful in getting a physical feel for how the transverse force is applied
- Explanation …





EMFF Vehicle Conceptual Model

- In the Far Field, Dipoles add as vectors
- Each vehicle will have 3 orthogonal electromagnetic coils
  - These will act as dipole vector components, and allow the magnetic dipole to be created in any direction
- Steering the dipoles electronically will decouple them from the spacecraft rotational dynamics
- A reaction wheel assembly with 3 orthogonal wheels provides counter torques to maintain attitude



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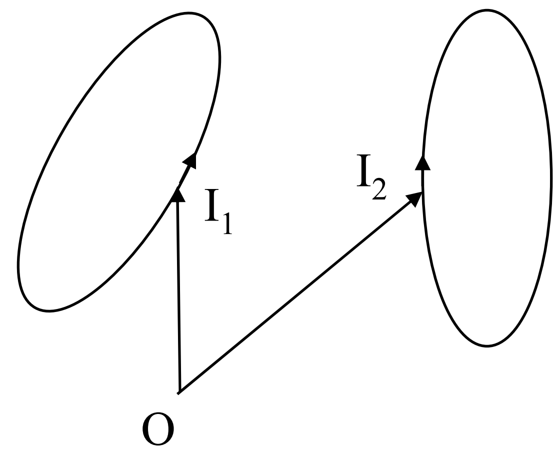
Magnetic Dipole Approximation

- The interaction force between two arbitrary magnetic circuits is given by the Law of Biot and Savart

$$\vec{F}_2 = \frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \oint_2 \frac{d\vec{l}_2 \times [d\vec{l}_1 \times (\vec{r}_2 - \vec{r}_1)]}{|\vec{r}_2 - \vec{r}_1|^3}$$



- In general, this is difficult to solve, except for cases of special symmetry
- Instead, at distances far from one of the circuits, the magnetic field can be approximated as a dipole

$$\vec{B} = \frac{\mu_o}{4\pi}\left[3\frac{(\vec{\mu}_1 \cdot \vec{r})}{r^5}\vec{r} - \frac{\vec{\mu}_1}{r^3}\right] = \frac{\mu_0}{4\pi}\left(\frac{\mu_1}{r^3}\right)\left[3(\hat{\mu}_1 \cdot \hat{r})\hat{r} - \hat{\mu}_1\right]$$

where its dipole strength μ_1 is given by the product of the total current around the loop (Amp-turns) and the area enclosed

Dipole-Dipole Interaction

- Just as an idealized electric charge in an external electric field can be assigned a scalar potential, so can an idealized magnetic dipole in a static external magnetic field, by taking the inner product of the two

$$U = -\vec{\mu}_2 \cdot \vec{B}$$

- Continuing the analogy, the force on the dipole is simply found by taking the negative potential gradient with respect to position coordinates

$$F = -\nabla_r U = \nabla_r (\vec{\mu}_2 \cdot \vec{B}) = \vec{\mu}_2 \cdot \nabla_r \vec{B}$$

- In a similar manner, taking the gradient with respect to angle will give the torque experienced by the dipole

$$T = -\nabla_\theta U = \vec{\mu}_2 \times \vec{B}$$

- Since the Force results from taking a gradient with respect to position, and the Torque does not, the scaling laws for the two are given as

$$|F| \sim \frac{3}{2\pi} \mu_0 \frac{\mu_1 \mu_2}{s^4} \qquad |T| \sim \frac{3}{4\pi} \mu_0 \frac{\mu_1 \mu_2}{s^3}$$

How Far Apart Will They Work?

- Writing the force in terms of the coil radius (*R*), separation distance (*s*) and total loop current (I_T), the force scales as

$$F \sim \frac{3\pi}{2}\mu_0 I_T^2 \left(\frac{R}{s}\right)^4$$

- We see that for a given coil current, the system scales 'photographically', meaning that two systems with the same loop current that are simply scaled versions of one another will have the same force
- For design, it is of interest to re-write in terms of coil mass and radius, and physical constants:

$$F \sim \frac{3\pi}{2}\mu_0 \left(\frac{M_C I_C}{2\pi R}\right)^2 \left(\frac{R}{s}\right)^4 = \frac{3}{2}(10^{-7})\left(\frac{I_C}{\rho}\right)^2 (M_C R_C)^2 \frac{1}{s^4}$$

- The current state-of-the-art HTS wire has a value of $\left(\frac{I_C}{\rho}\right) = 14{,}444\ A-m/kg$

And the product of coil mass and radius becomes the design parameter.

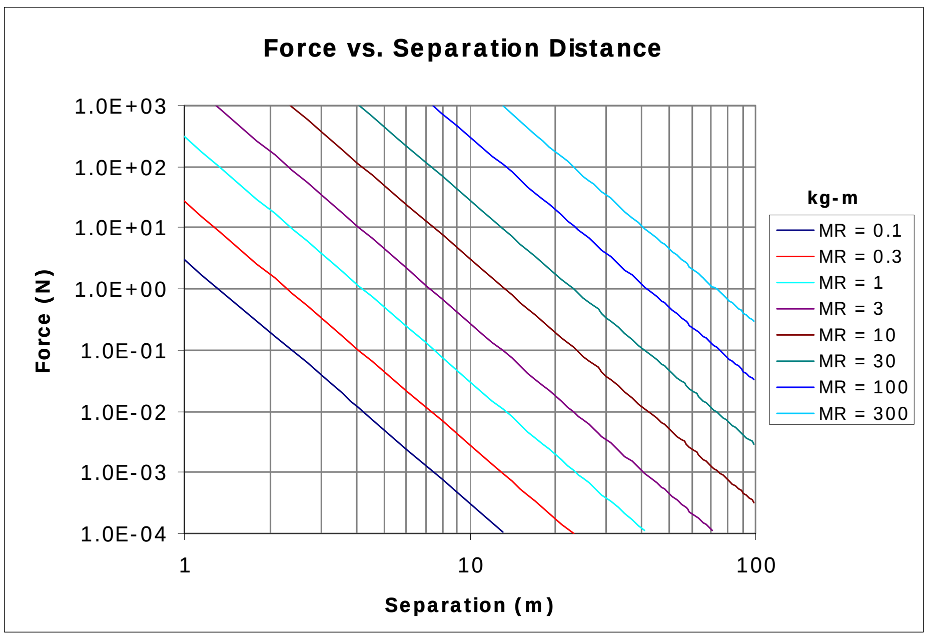
More of 'How Far apart will they work'?

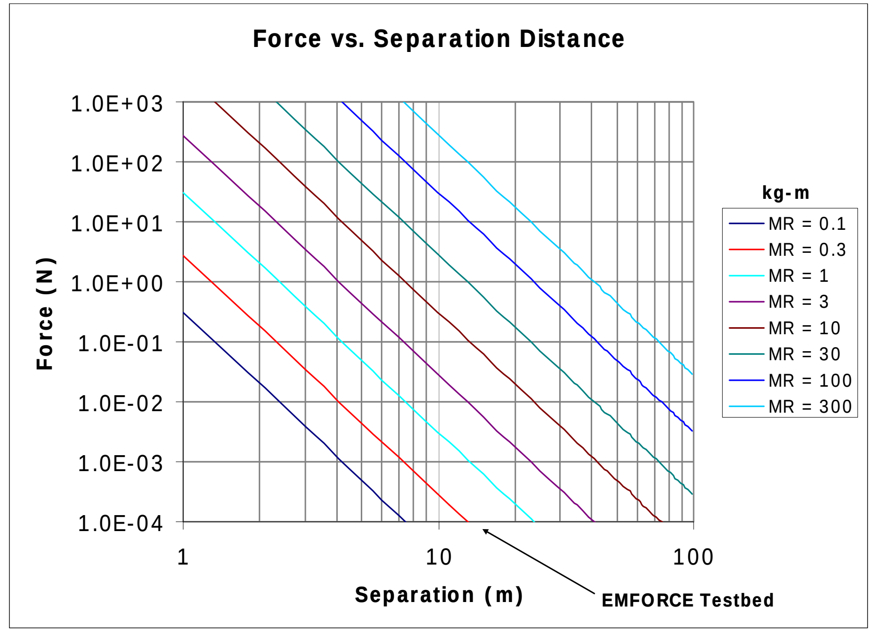
With further simplification:

$$F \sim 31.2\,(M_C R_C)^2 \frac{1}{s^4}$$

The graph to the right shows a family of curves for various products of M_C and R_C

$$\frac{3}{2}(10^{-7})\left(\frac{I_C}{\rho}\right)^2 = 312\ \frac{m^3}{kg\text{-}s^2}$$





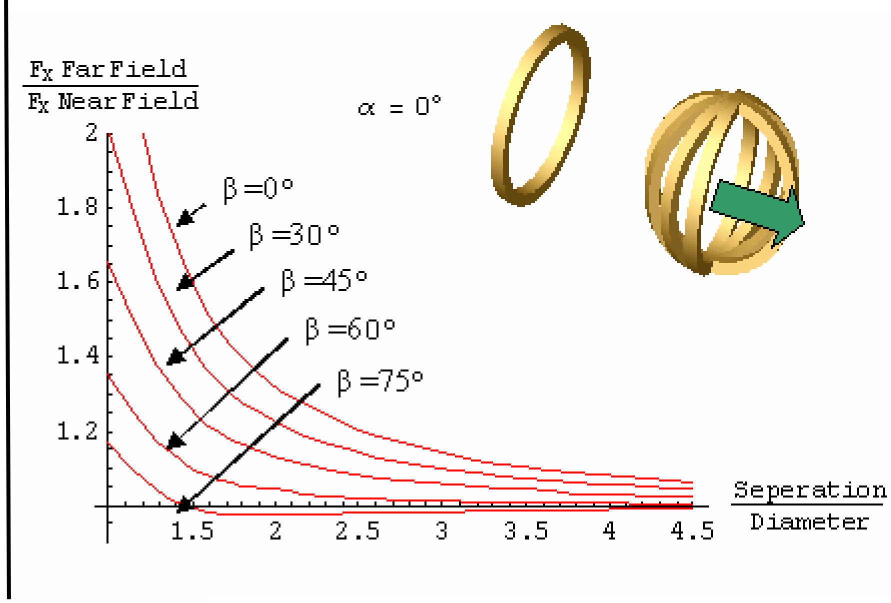
$$\frac{3}{2}(10^{-7})\left(\frac{I_C}{\rho}\right)^2 = 31.2\ \frac{m^3}{kg\text{-}s^2}$$

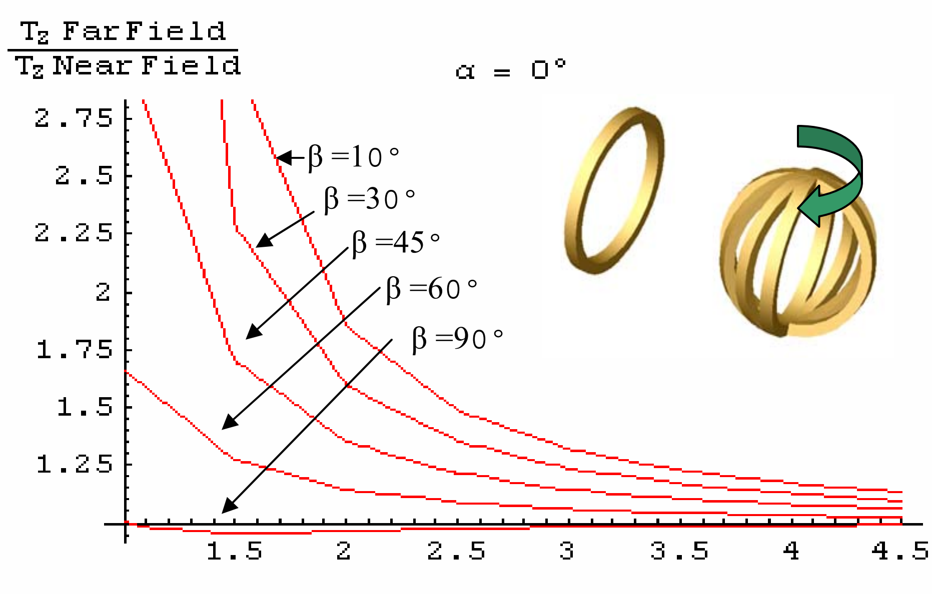
Example:

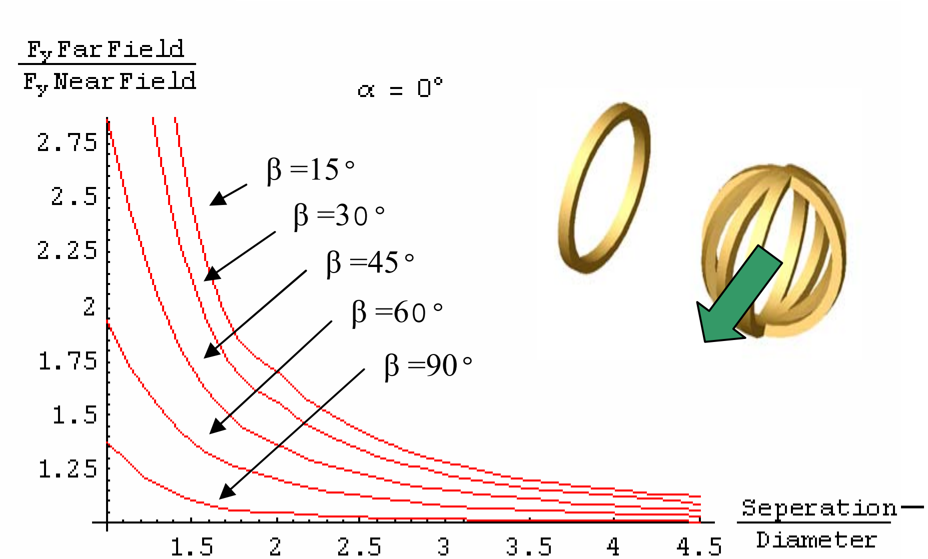
- 300 kg satellite, 2 m across, needs 10 mN of thrust, want M_C < 30 kg
- EMFF effective up to 40 meters

Far Field/Near Field Comparison

- The far field model does not work in the near field
- (Separation/Distance)>10 to be within 10%
  - Some configurations are more accurate
- A better model is needed for near-field motion since most mission applications will work in or near the edge of the near field
  - For TPF, (s/d) ~ 3 - 6







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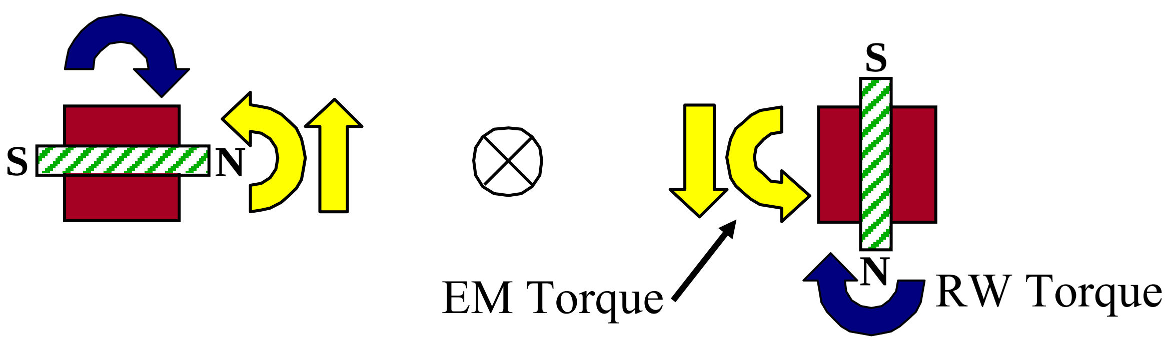
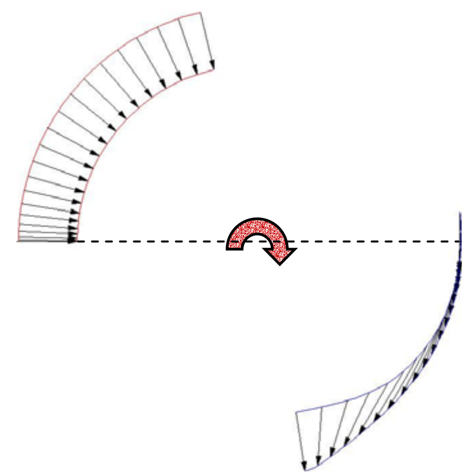
2-D Dynamics of Spin-Up

- Spin-up/spin-down
  - Spin-up from “static” baseline to rotating cluster for u-v plane filling
  - Spin-down to baseline that can be reoriented to a new target axis
- Electromagnets exert forces/torques on each other
  - Equal and opposite “shearing” forces
  - Torques in the same direction
- Reaction wheels (RW) are used to counteract EM torques
  - Initial torque caused by perpendicular-dipole orientation
  - Reaction wheels counter-torque to command EM orientation
  - Angular momentum conserved by shearing of the system



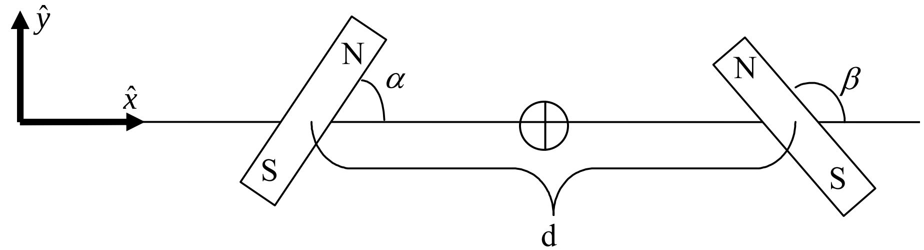


2-Satellite Spin-up

$$F_x = \frac{3}{4\pi}\frac{\mu_0\mu_A\mu_B}{d^4}(2\cos\alpha\cos\beta - \sin\alpha\sin\beta)$$

$$F_y = -\frac{3}{4\pi}\frac{\mu_0\mu_A\mu_B}{d^4}(\cos\alpha\sin\beta + \sin\alpha\cos\beta)$$

$$T_z = -\frac{1}{4\pi}\frac{\mu_0\mu_A\mu_B}{d^3}(\cos\alpha\sin\beta + 2\sin\alpha\cos\beta)$$



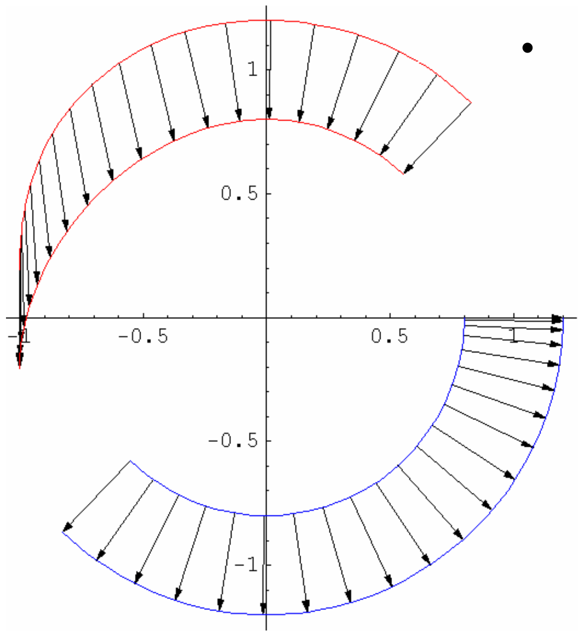
$$F_x = m\,a_{centripical} = m\omega^2 r$$

$$F_y = m\,\dot{\omega}\,r$$

$$\beta = 0$$

$$\mu_A\mu_B = \frac{32\pi}{3}\frac{m\,r^5\sqrt{4\dot{\omega}^2 + \omega^4}}{\mu_0}$$

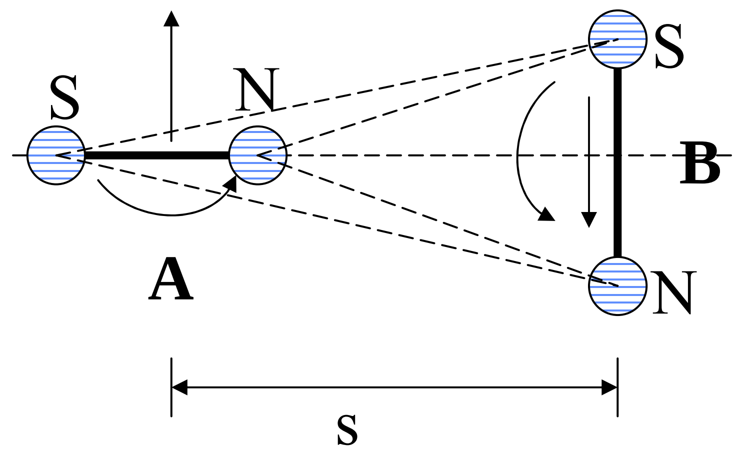
$$\alpha = -\cos^{-1}\left(\frac{\omega^2}{\sqrt{4\dot{\omega}^2 + \omega^4}}\right)$$

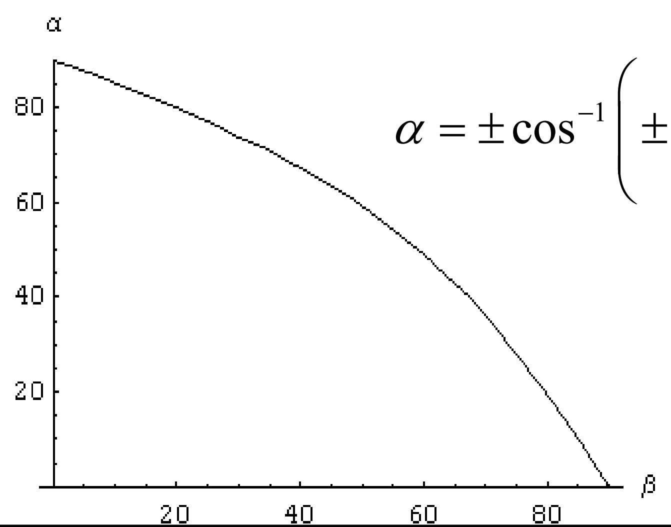


- 6 DOF (4 Translational, 2 Rotational)
- 4 DOF (2 Translational, 2 Rotational)
- 2 Reaction wheels control 2 Rotational DOF
- 2 dipole strengths and 2 dipole angles to control 2 translational degrees of freedom (relative motion)
  - 2 extra degrees of freedom.
  - Allows for many different spin-up configurations
  - Allows for different torque distribution
  - Become more complex with more satellites
  - Must solve non-linear system of equations

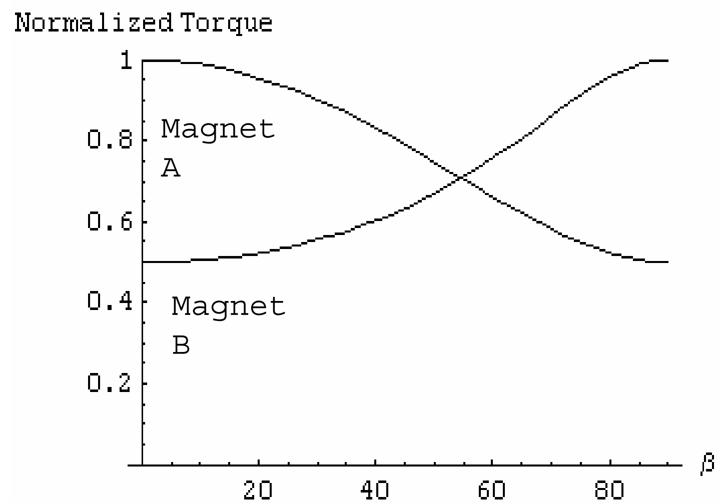
Torque Analysis

- Shear forces are produced when the dipole axes are not aligned.
- Torques are also produced when the shear forces are produced (Cosv. of angular mom.)
- The torques on each dipole is not usually equal
  - For the figure to the right $\frac{\tau_A}{\tau_B} = \frac{1}{2}$
- Even for pure shear forces, $(F_x = 0)$ one can arbitrarily pick one of the dipole angles.





$$\alpha = \pm\cos^{-1}\left(\pm\frac{\sqrt{2}\sin\beta}{\sqrt{5+3\cos(2\beta)}}\right)$$

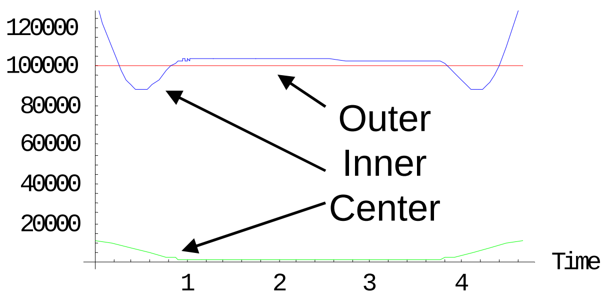


Satellite Formation Spin-Up

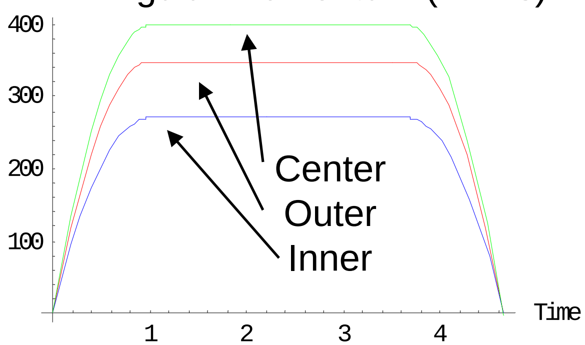
- Spin-up of complex formations can be achieved by utilizing magnetic dipoles.
- There are a number of possible combinations of magnet strengths and dipole configurations to achieve a given maneuver.
- These different configurations cause different distribution of angular momentum storage.

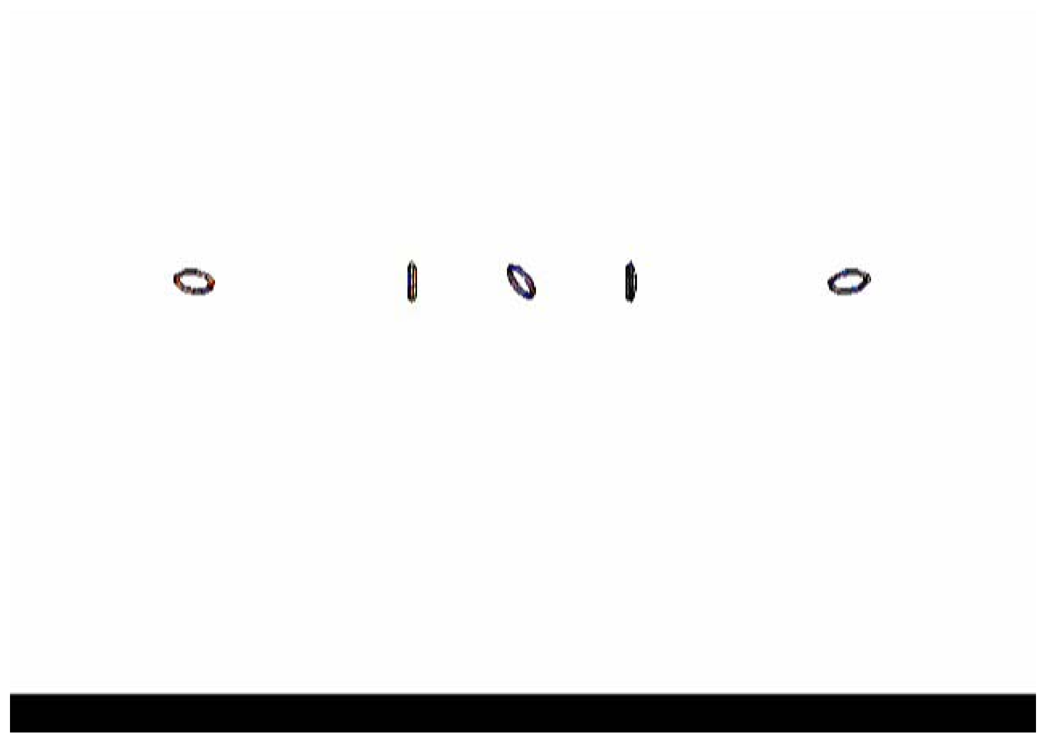
600 kg s/c, 75m diameter formation, 0.5 rev/hr

Magnetic Strength (A m^2)



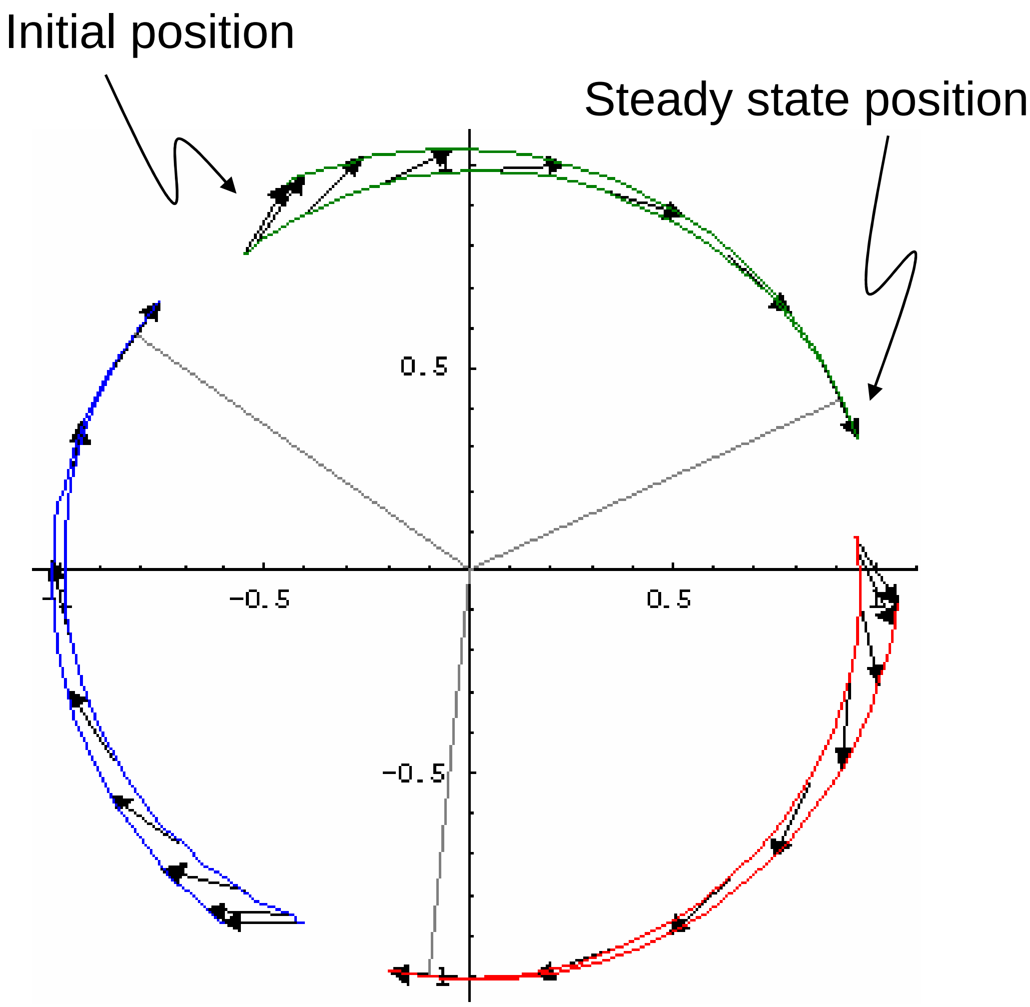
Angular Momentum (N m s)





Steady State Rotations

- Spin-up of formations are not restricted to linear arrays
- Configurations of any shape can be spun-up
- Shown here is a SPECS configuration of 3 satellites in an equilateral triangle.



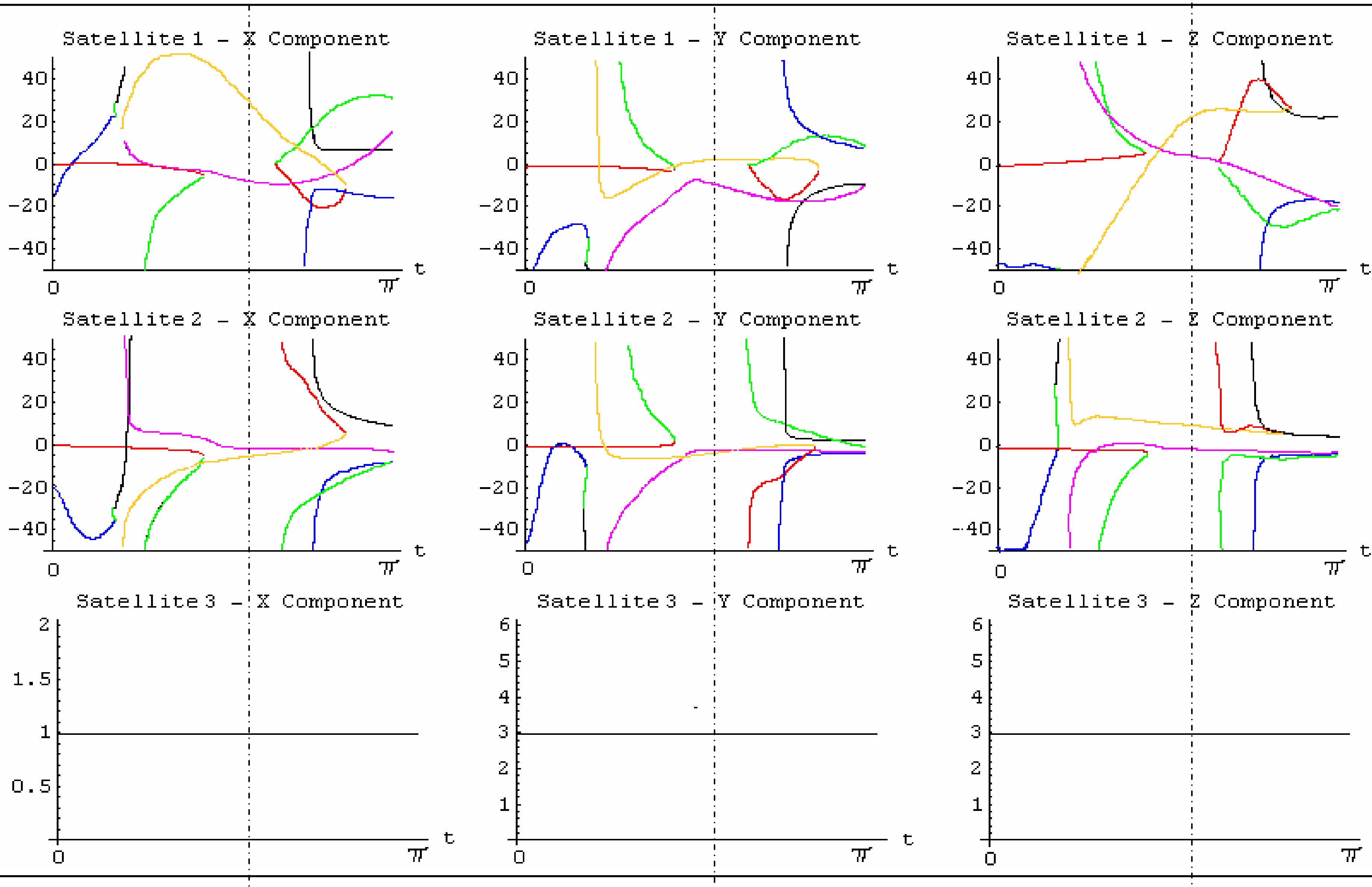
Solving the EOM

$$\vec{F}_A = \frac{3\mu_o}{4\pi}\left(-\frac{\vec{\mu}_A \bullet \vec{\mu}_B}{r^5}\vec{r} - \frac{\vec{\mu}_A \bullet \vec{r}}{r^5}\vec{\mu}_B - \frac{\vec{\mu}_B \bullet \vec{r}}{r^5}\vec{\mu}_A + 5\frac{(\vec{\mu}_A \bullet \vec{r})(\vec{\mu}_B \bullet \vec{r})}{r^7}\vec{r} \right)$$

- For a given instantaneous force profile, there are (3N-3) constraints (EOM), and 3N variables (Dipole strengths).
  - <u>This allows us to arbitrarily specify one vehicle's dipole</u>
  - Allows the user the freedom to control other aspects of the formation especially angular momentum distribution
  - For a specific choice of dipole, there are multiple solutions due to the non-linearity of the constraints

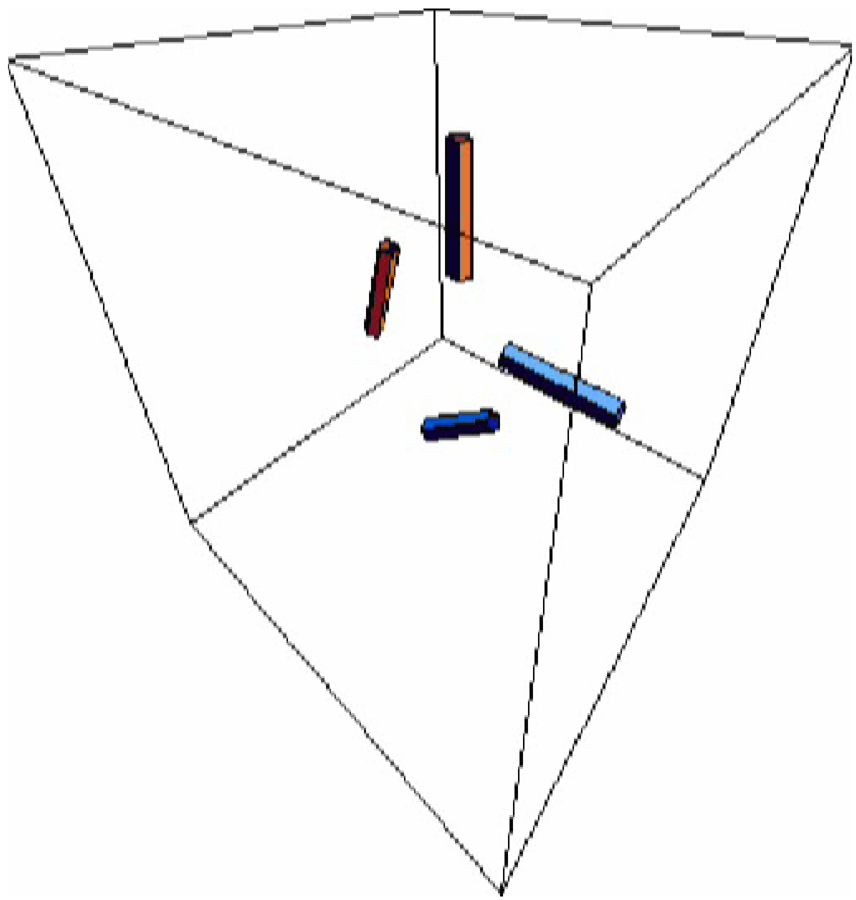
- To determine the required magnetic dipole strengths
  - Pick the magnetic dipole strengths for one vehicle
  - Set the first equation equal to the desired instantaneous force and solve for the remaining magnetic dipole strengths.
  - There will be multiple solutions. Pick the solution that is most favorable

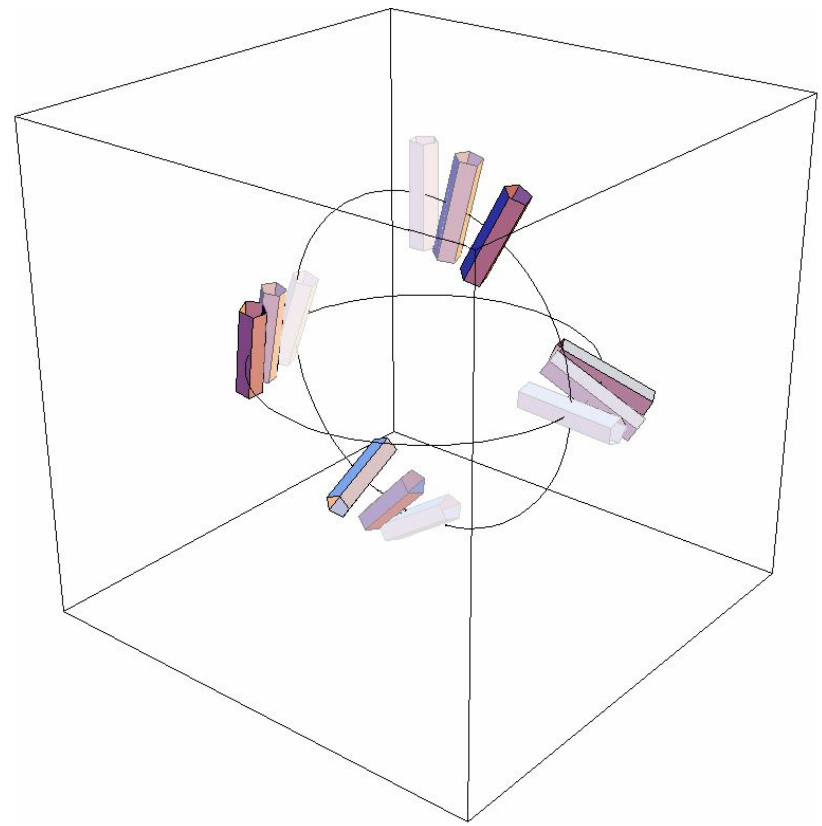
Multiple Solutions



3D Formations

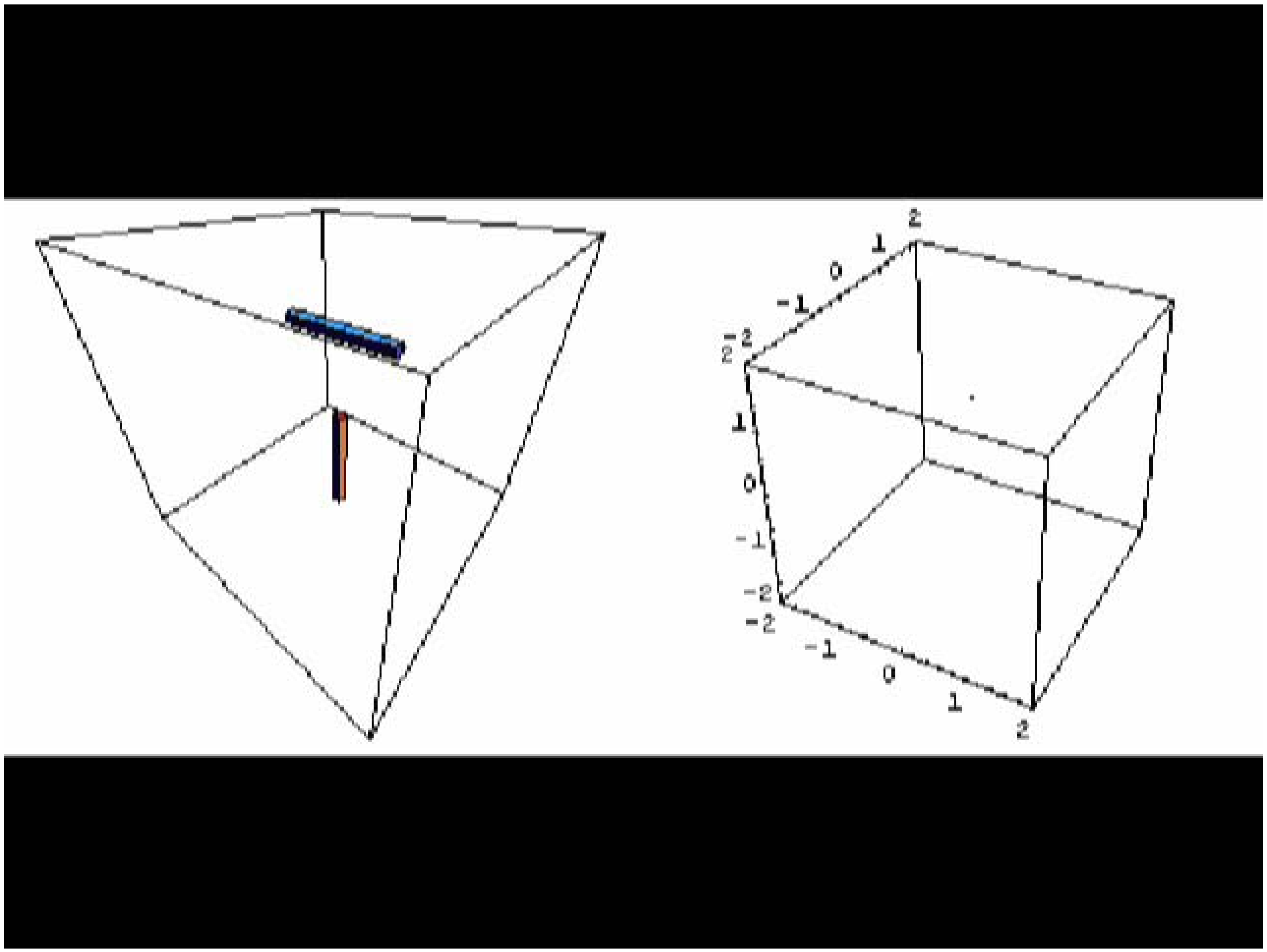
- We also have the ability to solve for complex 3D motion of satellites.





3D Formations

- Here is another example of a 3D configuration



Choosing the Free Dipole

- Choose the free dipole such that a cost function is optimized
  - Angular momentum distribution
  - Dipole strength distribution
  - Currently using Mathematica’s global minimization routine
    - Simulated Annealing
    - Genetic algorithms (Differential Evolution)
    - Nelder-Mead
    - Random Search

- Choose the free dipole based on a specific algorithm
  - Aligning with the Earth’s magnetic field
  - Favorable angular momentum distribution

Angular Momentum Management

- Being able to control the angular momentum gained by the individual satellites is crucial to the success of EMFF
- Because the torques and forces generated by EMFF are internal, there is no way to internally remove excess angular momentum from the system
  - Angular momentum can be transferred from one spacecraft to another
- Since EMFF systems do not employ thrusters, other innovative methods must be used to remove the excess angular momentum
  - The formation must interact with its environment
    - Using the Earth’s magnetic field
    - Using differential J2 forces

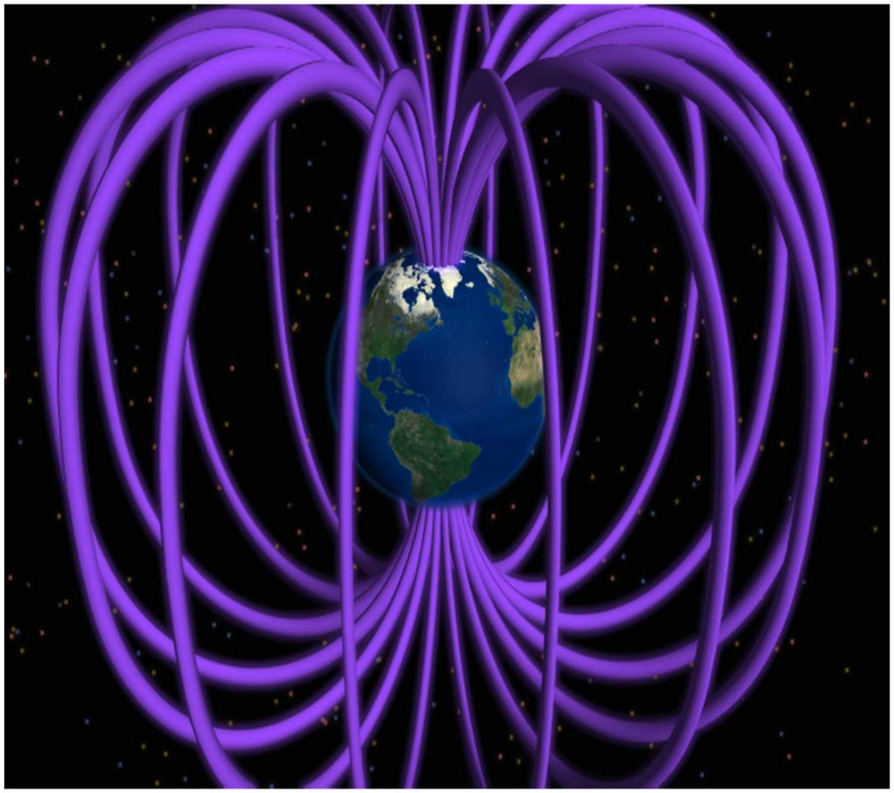
Earth's Magnetic Field

- Many formation flight missions will operate in LEO.

- The electromagnets will interact with the Earth's magnetic field producing unwanted forces and torques on the formation

- The Earth's magnetic field can be approximated by a large bar magnet with a magnetic dipole strength of $8*10^{22}$

- (EMFF Testbed ~ $2*10^{4}$)



Earth's Magnetic Field

- The Earth's Magnetic field produces an insignificant disturbance force, but a very significant disturbance torque, due to the scaling of force and torque

	Earth	Another Sat.
μ	$8*10^{22}$ Am^2	$5*10^{5}$ Am^2
d	> 6,378,000 m	2-100 m
$F \sim \mu_0 (\mu_1 \mu_2) / r^4$	$\sim 1*10^{-5}$ N	$\sim 1*10^{4}$-$2*10^{-3}$ N
$T \sim \mu_0 (\mu_1 \mu_2) / r^3$	$\sim 2*10^{1}$ Nm	$\sim 3*10^{4}$-$3*10^{-2}$ Nm

Possible Solutions for Dealing with the Earth's Magnetic Field

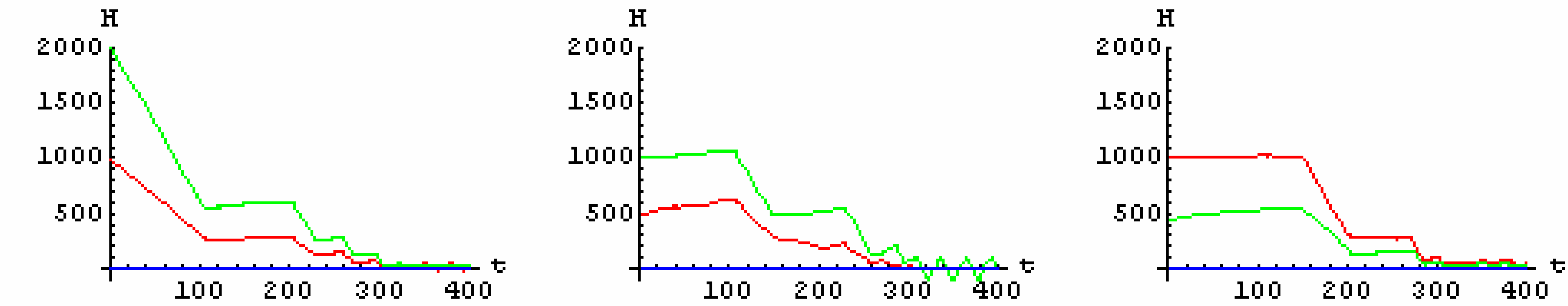
- Ignore the disturbance forces from the Earth's magnetic field on the formation as a whole
  - This frees up the arbitrary dipole, but disturbance forces are still accounted for.
- Periodically alternate the magnetic dipole directions, so that the accumulated torques average to zero
- Turn off all the satellites but one, and use the electromagnets like torque rods to dump the angular momentum
- Choose the arbitrary dipole wisely so that the total acquired angular momentum on the formation is zero
- Choose the arbitrary dipole wisely so that you can use the Earth as a dump for angular momentum.

The Earth as a Momentum Dump

- Use the Earth's dipole to our advantage by transferring angular momentum to the Earth
  - Already done for single spacecraft using torque rods
  - Can be expanded for use with satellite formations

- Strategy:
  - Pick a satellite to dump momentum
  - Turn up its dipole strength to maximum
  - Align the dipole to optimize momentum exchange
  - Solve the remaining dipoles for the required instantaneous forces
  - Once the required momentum has been dumped, pick another satellite that needs to dump momentum

Simulation Results



- Satellites are undergoing a specific forcing profile in the presence of the Earth's magnetic field
  - This way the satellites that are not dumping momentum are still being disturbed by the Earth's magnetic field.
- Each satellite starts off with excess angular momentum
- The satellite with the most excess momentum is selected for angular momentum dumping
- The formation is then maintained to have H<100

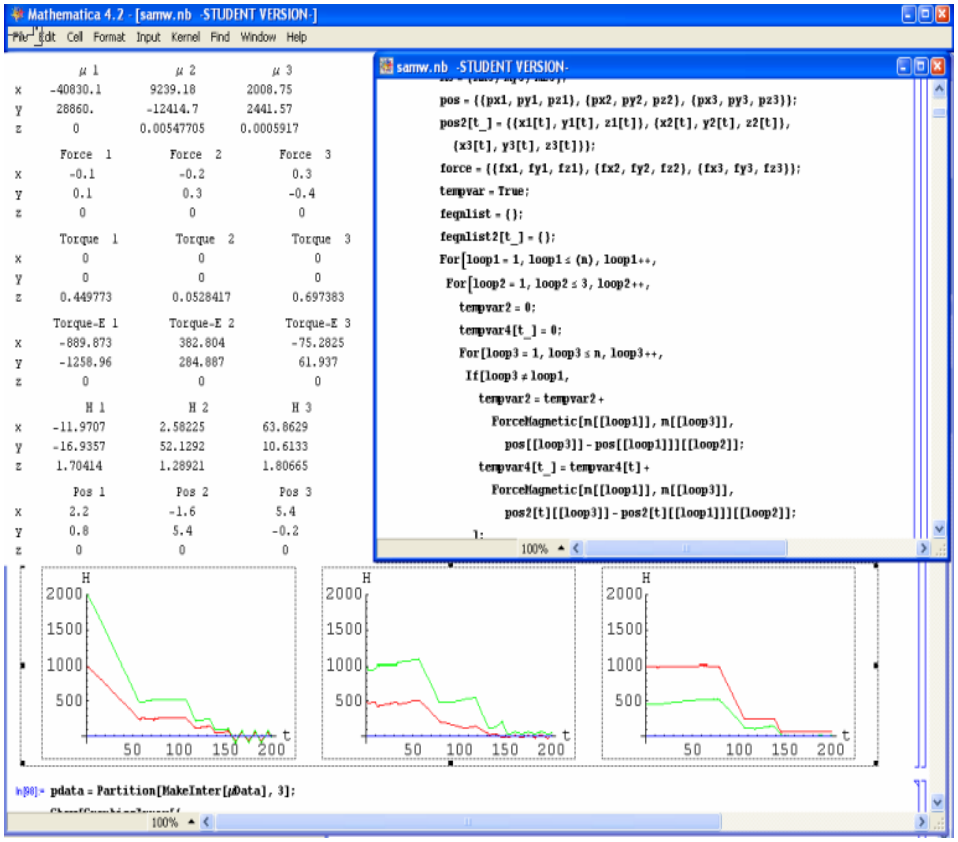
EMFF Simulator

- Currently designing a software simulator to test different angular momentum control schemes

- Built in Mathematica, it has the ability to provide MatLab style outputs

- It will have the ability to test control algorithms in the presence of the Earth's magnetic field or under the influence of the J2 disturbance force.

- Currently being used to verify angular momentum dumping algorithms in the presence of the Earth's magnetic field.



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  - Calibration
  - Movie
- **Space Hardware Design Issues**
  - Thermal Control
  - Power System Design
  - High B-Field Effects
- **Conclusions**

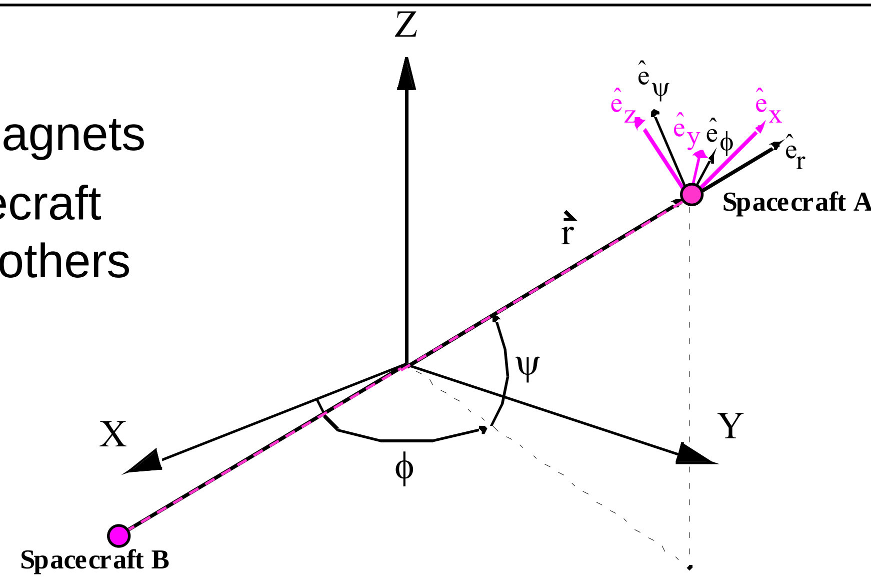
Objective : *Multi-Vehicle EMFF Analysis*

- **Motivation:**
  - Dynamic analyses must be performed to verify the stability and controllability of EMFF systems.

- **Objective:**
  - Derive the governing equations of motion for an EMFF system:
    - Analyze the relative displacements and rotations of the bodies.
    - Include the gyroscopic stiffening effect of spinning RWs on the vehicles.
  - Linearize the equations, and investigate the stability and controllability of the system.
  - Design a closed-loop linear controller for the system.
  - Perform a closed-loop time-simulation of the system to assess the model dynamics and control performance.
  - Experimentally validate the dynamics and control on a simplified hardware system.

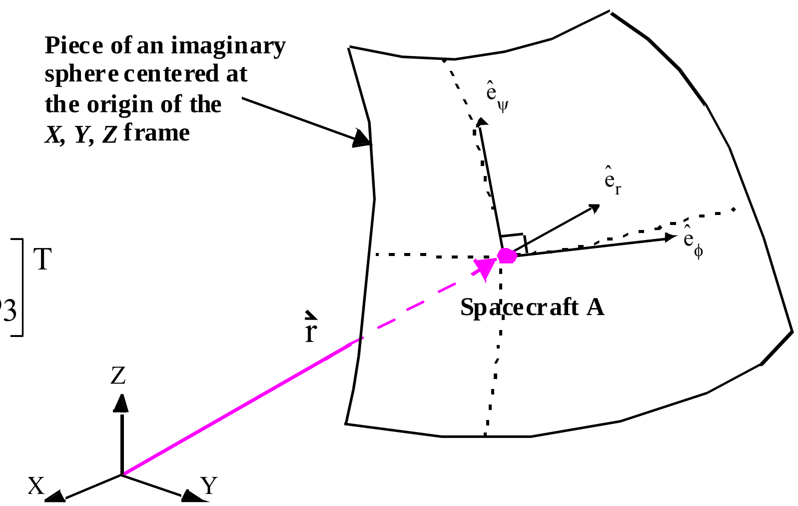
3-D Dynamics of 2-S/C EMFF Cluster

- Two-spacecraft array
  - Each has **three** orthogonal electromagnets
    - EM pointing toward other spacecraft carries bulk of centripetal load; others assist in disturbance rejection
  - Each has **three** orthogonal reaction wheels, used for system angular momentum storage and as attitude actuators



- State vector:

$$\boldsymbol{x} = \begin{bmatrix} r \ \phi \ \Psi \ \alpha_1 \ \alpha_2 \ \alpha_3 \ \beta_1 \ \beta_2 \ \beta_3 \ \dot{r} \ \dot{\phi} \ \dot{\Psi} \ \dot{\alpha}_1 \ \dot{\alpha}_2 \ \dot{\alpha}_3 \ \dot{\beta}_1 \ \dot{\beta}_2 \ \dot{\beta}_3 \end{bmatrix}^T$$



3-D Nonlinear Translational Equations

- Translational equations of motion for spacecraft A:

$$\ddot{\vec{r}} = \frac{1}{m}\vec{F}_{A/B} = \frac{1}{m}\left(\vec{F}_{A1/B1} + \vec{F}_{A1/B2} + \vec{F}_{A2/B1} + \vec{F}_{A2/B2}\right)$$

- In $\mathbf{e}_r$, $\mathbf{e}_\phi$, $\mathbf{e}_\psi$ components:

$$\ddot{\vec{r}} = \begin{Bmatrix} \ddot{r} - r\dot{\psi}^2 - r\dot{\phi}^2\cos^2\psi \\ 2\dot{r}\dot{\phi}\cos\psi + r\ddot{\phi}\cos\psi - 2r\dot{\phi}\dot{\psi}\sin\psi \\ 2\dot{r}\dot{\psi} + r\ddot{\psi} + r\dot{\phi}^2\sin\psi\cos\psi \end{Bmatrix}$$

- And the forcing terms are of the form:

$$\frac{\vec{F}_{A1/B1}}{m} = \frac{3\mu_0\mu_A\mu_B}{64\pi m r^4}\begin{Bmatrix} s\alpha_1 c\alpha_2 s\beta_1 c\beta_2 - 2c\alpha_1 c\alpha_2 c\beta_1 c\beta_2 + s\alpha_2 s\beta_2 \\ c\alpha_2 c\beta_2(s\alpha_1 c\beta_1 + s\beta_1 c\alpha_1) \\ -c\beta_1 c\beta_2 s\alpha_2 - c\alpha_1 c\alpha_2 s\beta_2 \end{Bmatrix}$$

3-D Nonlinear Rotational Equations

- Rotational equations of motion for spacecraft A:

$$\begin{bmatrix} I_{rr,s}+I_{rr,w} & 0 & 0 \\ 0 & I_{rr,s}+I_{rr,w} & 0 \\ 0 & 0 & I_{zz,s} \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_x \\ \ddot{\theta}_y \\ \ddot{\theta}_z \end{Bmatrix}_A + \begin{bmatrix} 0 & \Omega_{z,w} I_{zz,w} & 0 \\ -\Omega_{z,w} I_{zz,w} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_x \\ \dot{\theta}_y \\ \dot{\theta}_z \end{Bmatrix}_A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha_3 & s\alpha_3 \\ 0 & -s\alpha_3 & c\alpha_3 \end{bmatrix} \begin{bmatrix} c\alpha_2 & 0 & -s\alpha_2 \\ 0 & 1 & 0 \\ s\alpha_2 & 0 & c\alpha_2 \end{bmatrix} \begin{bmatrix} c\alpha_1 & s\alpha_1 & 0 \\ -s\alpha_1 & c\alpha_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} T_r \\ T_\phi \\ T_\psi \end{Bmatrix}_A$$

- $$\begin{Bmatrix} \dot{\theta}_x \\ \dot{\theta}_y \\ \dot{\theta}_z \end{Bmatrix}_A = \begin{Bmatrix} \dot{\alpha}_3 - \dot{\alpha}_1 s\alpha_2 \\ \dot{\alpha}_2 c\alpha_3 + \dot{\alpha}_1 c\alpha_2 s\alpha_3 \\ \dot{\alpha}_1 c\alpha_2 c\alpha_3 - \dot{\alpha}_2 s\alpha_3 \end{Bmatrix}$$.

$$\begin{Bmatrix} \ddot{\theta}_x \\ \ddot{\theta}_y \\ \ddot{\theta}_z \end{Bmatrix}_A = \begin{Bmatrix} \ddot{\alpha}_3 - \ddot{\alpha}_1 s\alpha_2 - \dot{\alpha}_1\dot{\alpha}_2 c\alpha_2 \\ \ddot{\alpha}_2 c\alpha_3 - \dot{\alpha}_2\dot{\alpha}_3 s\alpha_3 + \ddot{\alpha}_1 c\alpha_2 s\alpha_3 - \dot{\alpha}_1\dot{\alpha}_2 s\alpha_2 s\alpha_3 + \dot{\alpha}_1\dot{\alpha}_3 c\alpha_2 c\alpha_3 \\ \ddot{\alpha}_1 c\alpha_2 c\alpha_3 - \dot{\alpha}_1\dot{\alpha}_2 s\alpha_2 c\alpha_3 - \dot{\alpha}_1\dot{\alpha}_3 c\alpha_2 s\alpha_3 - \ddot{\alpha}_2 s\alpha_3 - \dot{\alpha}_2\dot{\alpha}_3 c\alpha_3 \end{Bmatrix}$$

$$\vec{T}_{A/B} = \vec{T}_{A1/B1} + \vec{T}_{A1/B2} + \vec{T}_{A2/B1} + \vec{T}_{A2/B2}$$

$$\begin{Bmatrix} T_r \\ T_\phi \\ T_\psi \end{Bmatrix}_{A1/B1} = \frac{-\mu_0 \mu_A \mu_B}{32\pi r^3} \begin{Bmatrix} s\alpha_2 s\beta_1 c\beta_2 - s\alpha_1 c\alpha_2 s\beta_2 \\ c\alpha_1 c\alpha_2 s\beta_2 + 2 s\alpha_2 c\beta_1 c\beta_2 \\ c\alpha_1 c\alpha_2 s\beta_1 c\beta_2 + 2 s\alpha_1 c\alpha_2 c\beta_1 c\beta_2 \end{Bmatrix}$$

Linearization: Nominal Point

- Conservation of Angular Momentum:

$$I_{zz,w}(\Omega_{z,w} + \dot{\phi}_0) + (I_{zz,s} + mr_0^2)\dot{\phi}_0 = 0$$

$$\Rightarrow \quad I_{zz,w}\Omega_{z,w} + mr_0^2\dot{\phi}_0 \approx 0$$

- Nominal State Trajectory:

$$\boldsymbol{x}_0 = \begin{bmatrix} r_0 & \phi_0 & \Psi_0 & \alpha_{1,0} & \alpha_{2,0} & \alpha_{3,0} & \beta_{1,0} & \beta_{2,0} & \beta_{3,0} & \dot{r}_0 & \dot{\phi}_0 & \dot{\Psi}_0 & \dot{\alpha}_{1,0} & \dot{\alpha}_{2,0} & \dot{\alpha}_{3,0} & \dot{\beta}_{1,0} & \dot{\beta}_{2,0} & \dot{\beta}_{3,0} \end{bmatrix}^T$$

$$= \begin{bmatrix} r_0 & \phi_0(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dot{\phi}_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

Motion in X-Y Plane: Linearized Equations

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & r_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & r_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & I_{zz,s} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & I_{rr,s}+I_{rr,w} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & I_{rr,s}+I_{rr,w} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & I_{zz,s} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{rr,s}+I_{rr,w} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{rr,s}+I_{rr,w}
\end{bmatrix}
\begin{Bmatrix} \Delta\ddot{r} \\ \Delta\ddot{\phi} \\ \Delta\ddot{\psi} \\ \Delta\ddot{\alpha}_1 \\ \Delta\ddot{\alpha}_2 \\ \Delta\ddot{\alpha}_3 \\ \Delta\ddot{\beta}_1 \\ \Delta\ddot{\beta}_2 \\ \Delta\ddot{\beta}_3 \end{Bmatrix}
+
\begin{bmatrix}
0 & -2r_0\dot{\phi}_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2\dot{\phi}_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & mr_0^2\dot{\phi}_0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -mr_0^2\dot{\phi}_0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & mr_0^2\dot{\phi}_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -mr_0^2\dot{\phi}_0 & 0
\end{bmatrix}
\begin{Bmatrix} \Delta\dot{r} \\ \Delta\dot{\phi} \\ \Delta\dot{\psi} \\ \Delta\dot{\alpha}_1 \\ \Delta\dot{\alpha}_2 \\ \Delta\dot{\alpha}_3 \\ \Delta\dot{\beta}_1 \\ \Delta\dot{\beta}_2 \\ \Delta\dot{\beta}_3 \end{Bmatrix}
$$

Gyroscopic Stiffening Terms

$$
+
\begin{bmatrix}
-5\dot{\phi}_0^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & c_1 & 0 & 0 & c_1 & 0 & 0 \\
0 & 0 & r_0\dot{\phi}_0^2 & 0 & -c_1 & 0 & 0 & -c_1 & 0 \\
0 & 0 & 0 & -2c_0 & 0 & 0 & -c_0 & 0 & 0 \\
0 & 0 & 0 & 0 & -2c_0 & 0 & 0 & -c_0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -c_0 & 0 & 0 & -2c_0 & 0 & 0 \\
0 & 0 & 0 & 0 & -c_0 & 0 & 0 & -2c_0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{Bmatrix} \Delta r \\ \Delta\phi \\ \Delta\psi \\ \Delta\alpha_1 \\ \Delta\alpha_2 \\ \Delta\alpha_3 \\ \Delta\beta_1 \\ \Delta\beta_2 \\ \Delta\beta_3 \end{Bmatrix}
=
\begin{bmatrix}
c_2 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{c_2}{2} & 0 & 0 & -\frac{c_2}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{c_2}{2} & 0 & 0 & -\frac{c_2}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2c_3 & 0 & 0 & c_3 & 0 & K_T & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2c_3 & 0 & 0 & -c_3 & 0 & K_T & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_T & 0 & 0 & 0 \\
0 & c_3 & 0 & 0 & 2c_3 & 0 & 0 & 0 & 0 & K_T & 0 & 0 \\
0 & 0 & -c_3 & 0 & 0 & -2c_3 & 0 & 0 & 0 & 0 & K_T & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_T
\end{bmatrix}
\begin{Bmatrix} \Delta\mu_{A1} \\ \Delta\mu_{A2} \\ \Delta\mu_{A3} \\ \Delta\mu_{B1} \\ \Delta\mu_{B2} \\ \Delta\mu_{B3} \\ \Delta i_{RW,A1} \\ \Delta i_{RW,A2} \\ \Delta i_{RW,A3} \\ \Delta i_{RW,B1} \\ \Delta i_{RW,B2} \\ \Delta i_{RW,B3} \end{Bmatrix}
$$

$$c_0 \equiv \frac{-mr_0^2\dot{\phi}_0^2}{3}$$

$$c_1 \equiv \frac{-r_0\dot{\phi}_0^2}{2}$$

$$c_2 \equiv -\dot{\phi}_0\sqrt{\frac{3\mu_0}{32\pi m r_0^3}}$$

EMFF Stability

- Full-state system (n=18) has eigenvalues:

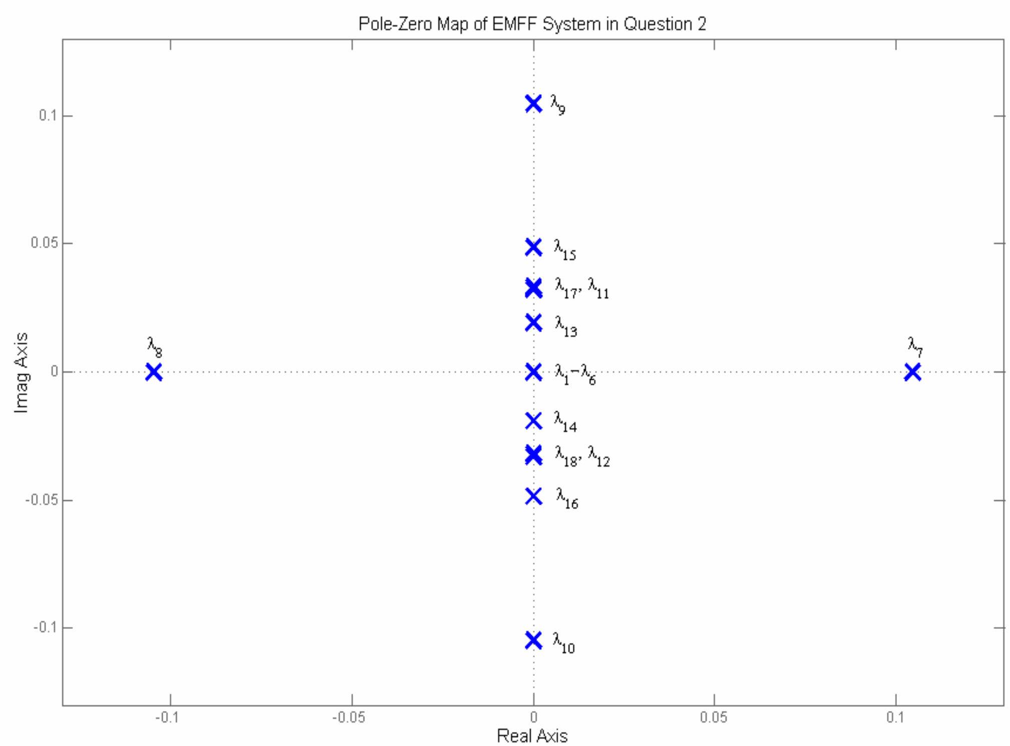
$$\lambda_{1,2,3,4,5,6} = 0 \qquad \lambda_{7,8} = \pm\dot{\phi}_0 \qquad \lambda_{9,10} = \pm i\dot{\phi}_0$$

$$\lambda_{11,12} = \pm i\frac{r_0\dot{\phi}_0}{(I_{rr,s}+I_{rr,w})}\sqrt{m\left(mr_0^2+\frac{I_{rr,s}+I_{rr,w}}{3}\right)} \qquad \lambda_{13,14} = \pm i\frac{r_0\dot{\phi}_0}{(I_{rr,s}+I_{rr,w})}\sqrt{m(mr_0^2+I_{rr,s}+I_{rr,w})}$$

$$\lambda_{15,16} = \pm ir_0\dot{\phi}_0\sqrt{\frac{m}{3I_{zz,s}}}$$

$$\lambda_{17,18} = \pm ir_0\dot{\phi}_0\sqrt{\frac{m}{I_{zz,s}}}$$

- Several poles on the imaginary axis and one unstable pole
- $\lambda_{7,8}$ at +/- array spin-rate
- Poles move away from origin as $\dot{\phi}_0$ increases



EMFF Controllability

- Current system is in 2nd order form:

$$M\ddot{\tilde{x}} + C\dot{\tilde{x}} + K\tilde{x} = Fu$$

- Place in 1st order form:

$$\dot{x} = \mathrm{A}x + \mathrm{B}u \qquad x = [\tilde{x} \quad \dot{\tilde{x}}]^{\mathrm{T}}$$

$$\mathrm{A} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \qquad \mathrm{B} = \begin{bmatrix} 0 \\ M^{-1}F \end{bmatrix}$$

- Form controllability matrix:

$$C = [\mathrm{B} \quad \mathrm{AB} \quad \mathrm{A}^2\mathrm{B} \quad \ldots \quad \mathrm{A}^{n-1}\mathrm{B}]$$

- **System is fully controllable because *C* has full rank**

$$\mathbf{rank}\,(C) = \mathbf{18} = n \qquad n : \text{number of states}$$

EMFF Linear Controller Design

- From state-space equation of motion: $\dot{x} = \mathrm{A}x + \mathrm{B}u$
- Form the LQR cost function: $J = \int_0^\infty \left[\mathbf{x}^T R_{xx} \mathbf{x} + \mathbf{u}^T R_{uu} \mathbf{u}\right] dt$

R_{xx}: state penalty matrix R_{uu}: control penalty matrix

Choose relative state and control penalties:

- $\Delta r : 10$ $\Delta \dot{r} : 5$ $\Delta \phi : 10^{-15}$ $\Delta \dot{\phi} : 3$ $\Delta \psi : 1$ $\Delta \dot{\psi} : 1$
- All Euler angles and their derivatives : 1
- All electromagnets, all reaction wheels : 1

- The cost, J, is minimized when:

$$0 = R_{xx} + PA + A^T P - PBR_{uu}^{-1} B^T P \longrightarrow \mathbf{u} = -R_{uu}^{-1} B^T P \mathbf{x} = -K\mathbf{x}$$

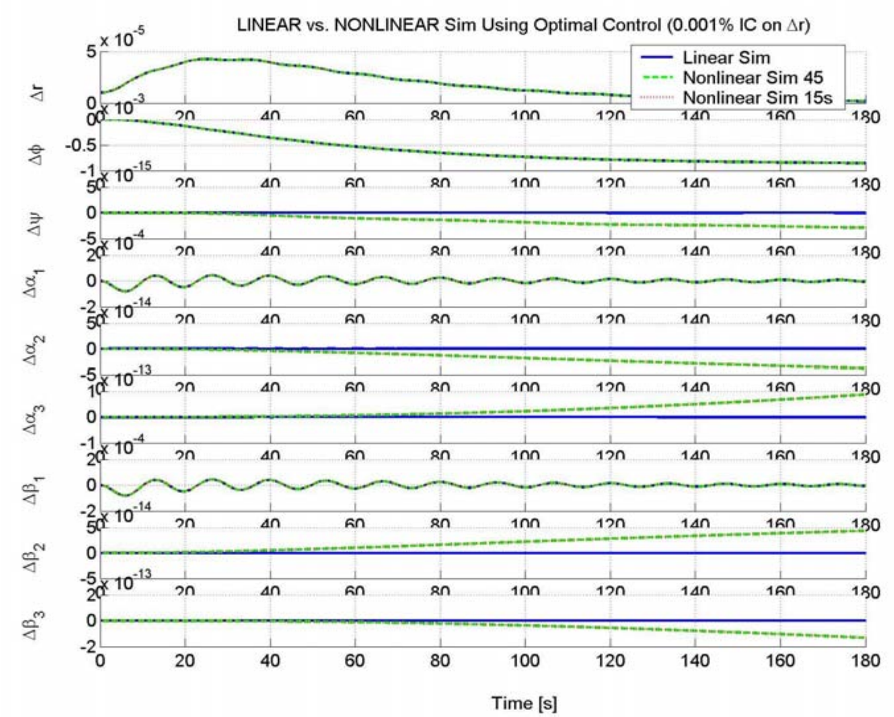
Algebraic Ricatti Equation (A.R.E.)

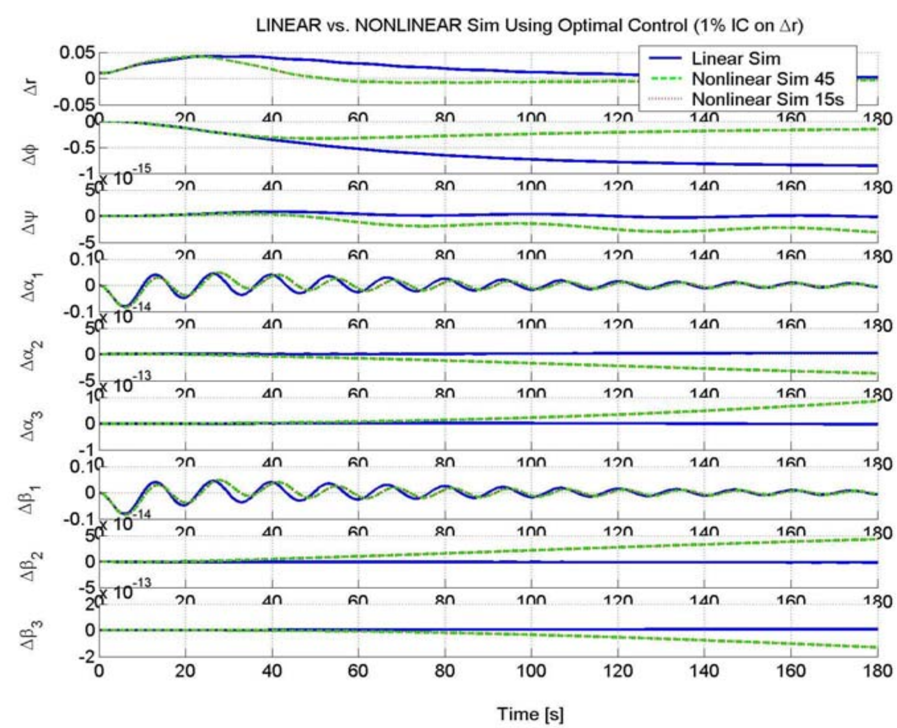
$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} = \left[A - BK\right]\mathbf{x} = A_{CL}\mathbf{x}$$

Simulation of EMFF Dynamics

- Closed-loop time simulations were performed of both the nonlinear and linearized equations of motion
  - Both employ the same linear feedback controller
- "Free vibration" response was investigated
  - Initial condition : deviation from nominal state of one or more degrees of freedom (Δr in the results shown here)
  - Closed-loop response to perturbed initial condition is simulated
  - Perhaps offers more insight than simulating response to random disturbances
- Results demonstrate:
  - the range in which the *linearized equations* are valid
  - the range in which *linear control* is sufficient
  - the importance of the relative control penalties chosen for the various degrees of freedom

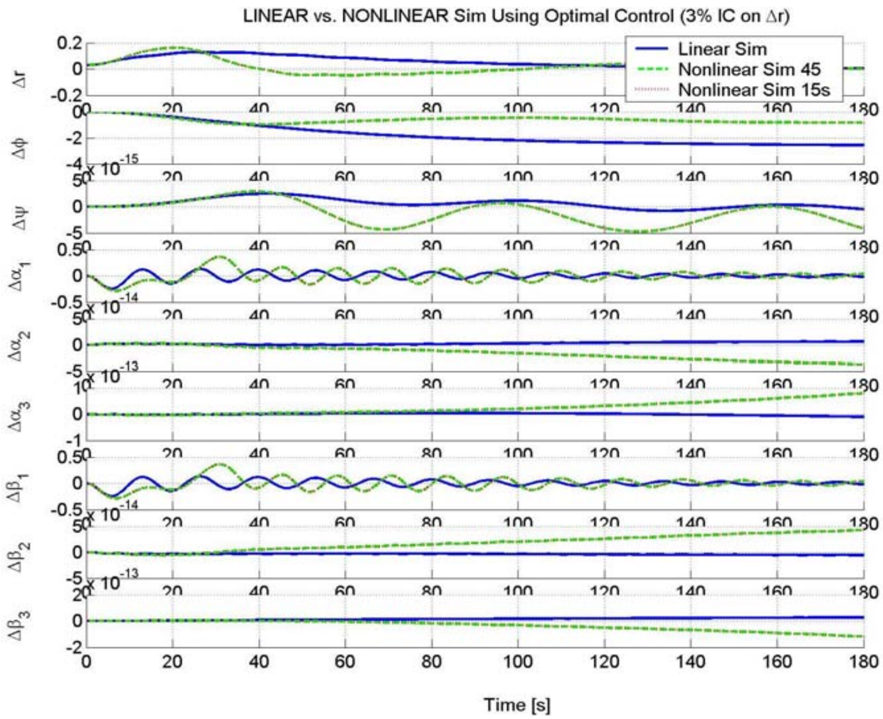
Simulation of EMFF Dynamics Results (I)

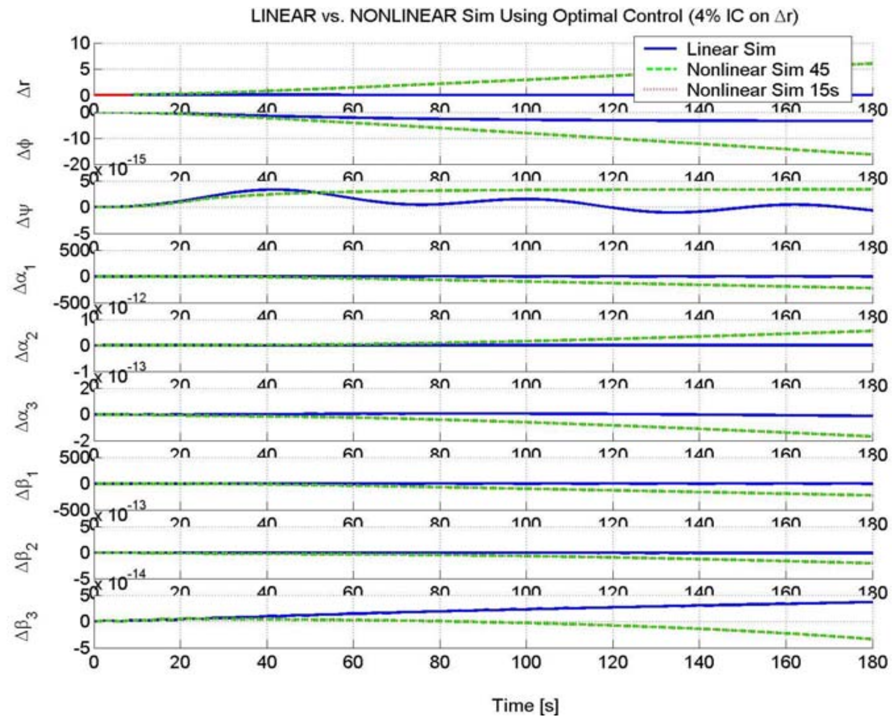




- Initial conditions: 0.001% deviation from nominal array radius
  - Simulations of nonlinear and linearized equations are identical, except for small numerical error in angles $\Delta\psi$, $\Delta\alpha_2$, $\Delta\alpha_3$, $\Delta\beta_2$, $\Delta\beta_3$
  - Both use linear controller
- Initial conditions: 1% deviation from nominal array radius
  - Nonlinear and linear simulations diverge
  - System remains stable in both simulations

Simulation of EMFF Dynamics Results (II)

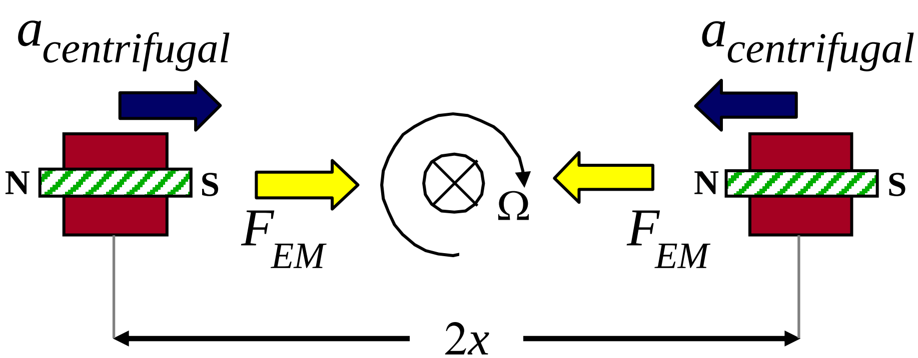




- Initial conditions: 3% deviation from nominal array radius
  - Radial separation remains stable
  - Elevation angle of array may go unstable (probably numerical error).
  - Check by increasing relative penalty on $\Delta\psi$ and redesigning controller.
- Initial conditions: 4% deviation from nominal array radius
  - Divergence of radial separation shows linear control not sufficient in this case.
    - Redesign with greater penalty on Δr?
    - Investigate nonlinear control techniques?
  - Linear simulation does not capture divergence of dynamics.

Simplified System: Steady-State Spin

- 1-D simplification of linearized 3-D dynamics
- Constant spin rate for data collection
- Relative radial position maintenance: disturbance rejection



$$a_{cent} = \Omega^2 x$$

$$F_{EM} = \frac{c\mu^2}{(2x)^4}, \quad c = \frac{3\mu_0}{2\pi}$$

$$\ddot{x} - a_{cent} = -\frac{F_{EM}}{m}$$

$$\ddot{x} = \Omega^2 x - \frac{c\mu^2}{16mx^4}$$

Perturbed Dynamics of Steady-State Spin

- **Perturbation Analysis:**

$$x = x_0 + \delta x \ , \quad \mu = \mu_{avg} + \delta\mu$$

$$\ddot{x}_0 + \delta\ddot{x} = \Omega^2\left(x_0 + \delta x\right) - \frac{c\left(\mu_{avg} + \delta\mu\right)^2}{16m\left(x_0 + \delta x\right)^4}$$

$$\mu_{avg}{}^2 = \frac{16m\Omega^2 x_0^5}{c}$$

Steady-State Control

$$\frac{\delta\ddot{x}}{x_0} - \Omega^2\frac{\delta x}{x_0} = -2\Omega^2\frac{\delta\mu}{\mu_{avg}}$$

Perturbation Equation

- Use binomial formula to expand terms
- Neglect H.O.T.
- Solve for S.S. Control when $\ddot{x} = 0$

$$\begin{bmatrix} \frac{\delta\dot{x}}{x_0} \\ \frac{\delta\ddot{x}}{x_0} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \Omega^2 & 0 \end{bmatrix} \begin{bmatrix} \frac{\delta x}{x_0} \\ \frac{\delta\dot{x}}{x_0} \end{bmatrix} + \begin{bmatrix} 0 \\ -2\Omega^2 \end{bmatrix} \frac{\delta\mu}{\mu_{avg}}$$

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$

→

Unstable dynamics:

$$s_{1,2} = \pm\Omega$$

*** Same as $\lambda_{7,8}$ in 3-D EMFF system analysis**

Linear Control Design

- Follow same control design process as for full-state, 3-D system:

$$J = \int_0^\infty \left[\mathbf{x}^T R_{xx}\mathbf{x} + \mathbf{u}^T R_{uu}\mathbf{u}\right]dt$$

$$\mathbf{u} = -R_{uu}^{-1}B^T P\mathbf{x} = -K\mathbf{x}$$

- Select state and control penalties:

$$R_{xx} = \begin{bmatrix} \lambda & 0 \\ 0 & 0 \end{bmatrix} \qquad R_{uu} = \rho$$

- Solve the A.R.E. analytically by enforcing
that P must be positive semidefinite:

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} \geq 0$$

- The displacement and velocity **feedback gains** are then:

$$K = R_{uu}^{-1}B^T P = \frac{2\Omega^2}{\rho}\begin{bmatrix} P_{12} & P_{22} \end{bmatrix}$$

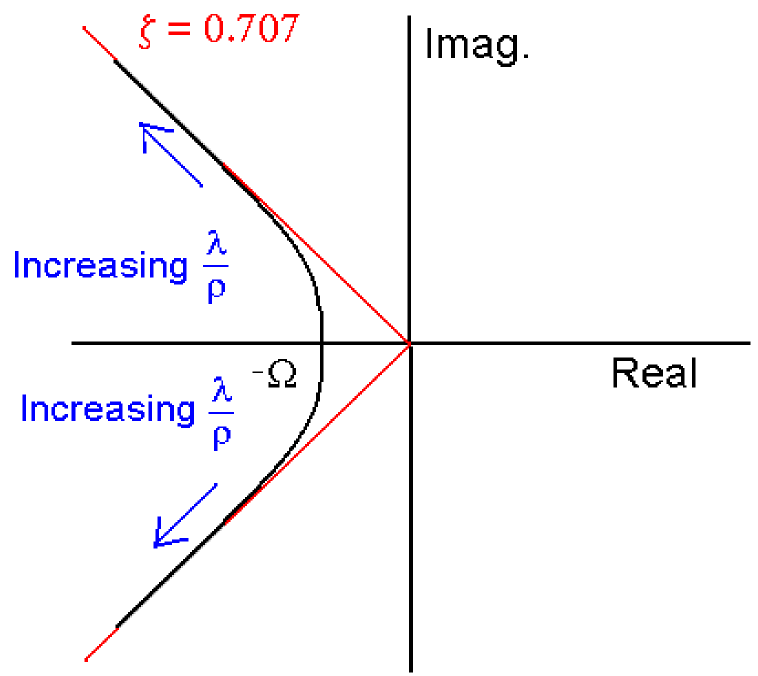
State-Space Analysis

- Now solve for the closed-loop dynamic matrix, where:

$$\mathbf{u} = -K\mathbf{x} \longrightarrow \dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} = \left[A - BK\right]\mathbf{x} = A_{CL}\mathbf{x}$$

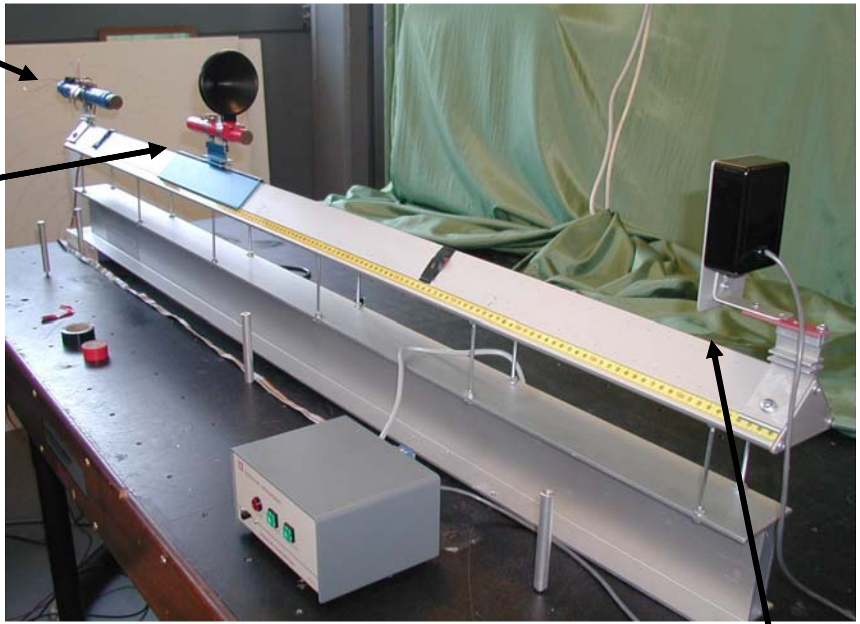
- Evaluate as $\frac{\lambda}{\rho}$ increases from $0 \rightarrow \infty$

- The closed-loop poles for the most efficient controller lie along this curve.

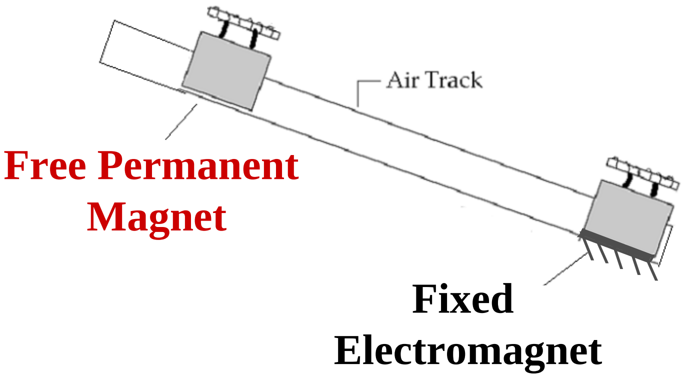


Experimental Validation: 1-D Airtrack

- Nearly frictionless 1-dimensional airtrack
- Can be set up in a **stable** or **unstable** configuration, depending on the tilt angle
- Unstable mode has dynamics nearly identical to a 1-DOF steady-state spinning cluster!
  - Closing the loop on the unstable configuration will demonstrate an ability to control systems such as the steady-state spinning cluster.



Experimental Results: Stable Airtrack

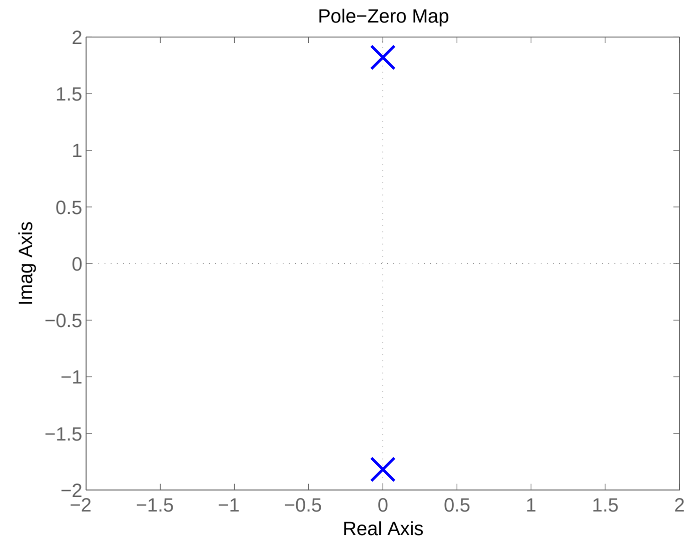


Stable poles:

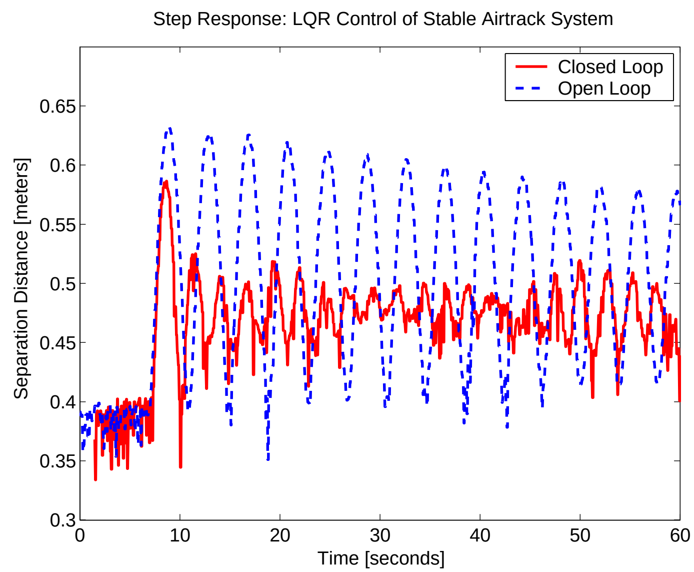
$$s_{1,2} = \pm i\sqrt{\frac{6\mu_0\mu_p}{\pi m x_0^5}}$$

Optimal gains:

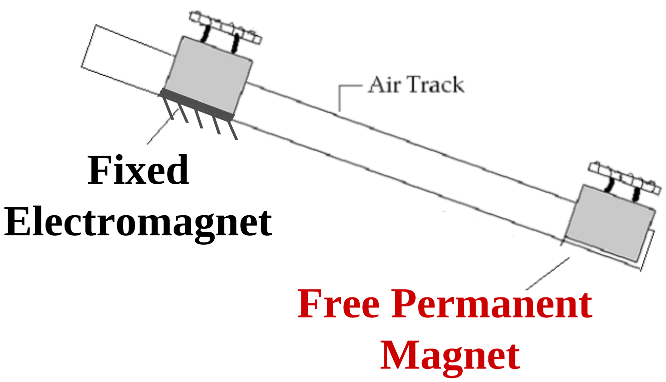
$$K = \begin{bmatrix} 11.74 & 4.33 \end{bmatrix}$$



- Similar linearization, state-space analysis, and **LQR control design** to steady-state spin system
- Open-loop step response
  - Very light damping means poles are nearly on the imaginary axis, as expected
- Closed-loop step response has reduced overshoot and increased damping



Experimental Results: Unstable Airtrack

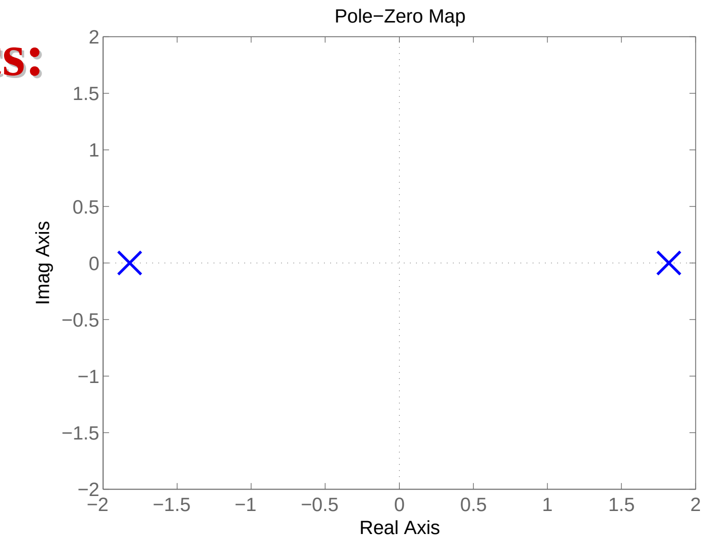


Unstable dynamics:

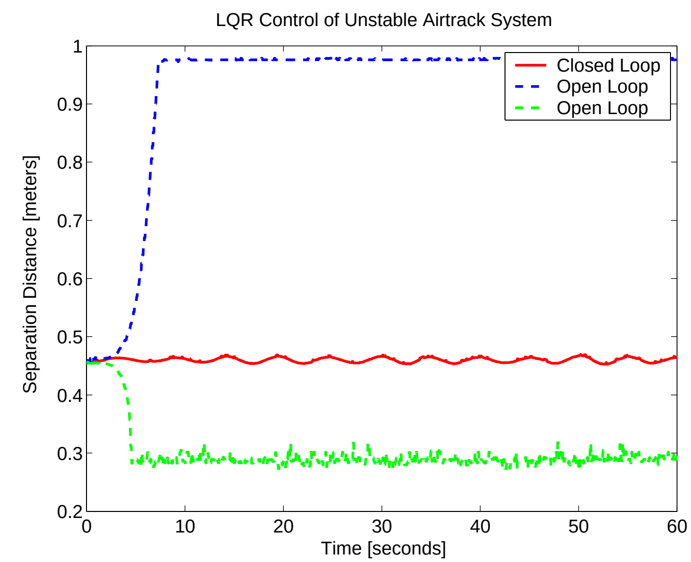
$$s_{1,2} = \pm\sqrt{\frac{6\mu_0\mu_p}{\pi m x_0^5}}$$

Optimal gains:

$$K = -\begin{bmatrix} 2.56 & 0.88 \end{bmatrix}$$



- Similar dynamics and **control design** to steady-state spin and stable-airtrack
- Open-loop response is divergent
- Closed-loop response is stable!
- Stabilizing this system means we should be able to perform steady-state control and disturbance rejection for a spinning cluster!



Video: Control of Unstable Airtrack

- Open-loop response is divergent.
  - Constant current is applied to EM
  - Magnets diverge from steady-state separation distance
    - Fall apart if disturbed one way
    - Come together if disturbed the other way

- Closed-loop response is stable!
  - Oscillates at about ~0.2 Hz
  - Maximum displacement from steady-state location is ~1 cm
  - Performance limitations due to model uncertainty and amplifier saturation

Control Summary

- Modeled the dynamics of a two-vehicle EMFF cluster
  - Nonlinear, unstable dynamics
  - Linearized dynamics about a nominal trajectory (steady-state spin)
  - **Stability: 3-D system has six poles at the origin, ten poles along the imaginary axis, and a stable/unstable pair of poles at the array spin-rate**
  - **Controllability: System is fully controllable with 3 electromagnets and 3 reaction wheels per vehicle**
- Simulated two-vehicle EMFF closed-loop dynamics
  - Demonstrated stabilization of unstable nonlinear dynamics using linear control
  - We can investigate for future systems:
    - whether linear control is sufficient for a given configuration
    - what the "allowable" disturbances are from the nominal state
    - how the relative state and control penalties may improve the closed-loop behavior
- Validated EMFF dynamics and closed-loop control on simplified hardware system
  - Airtrack: stable and unstable configurations (1-DOF)
  - Demonstrated stabilization of an unstable system with dynamics similar to an EMFF array undergoing steady-state-spin

Outline

- **Motivation**
- **Fundamental Principles**
  - Governing Equations
  - Trajectory Mechanics
  - Stability and Control
- **Mission Applicability**
  - Sparse Arrays
  - Filled Apertures
  - Other Proximity Operations
- **Mission Analyses**
  - Sparse Arrays
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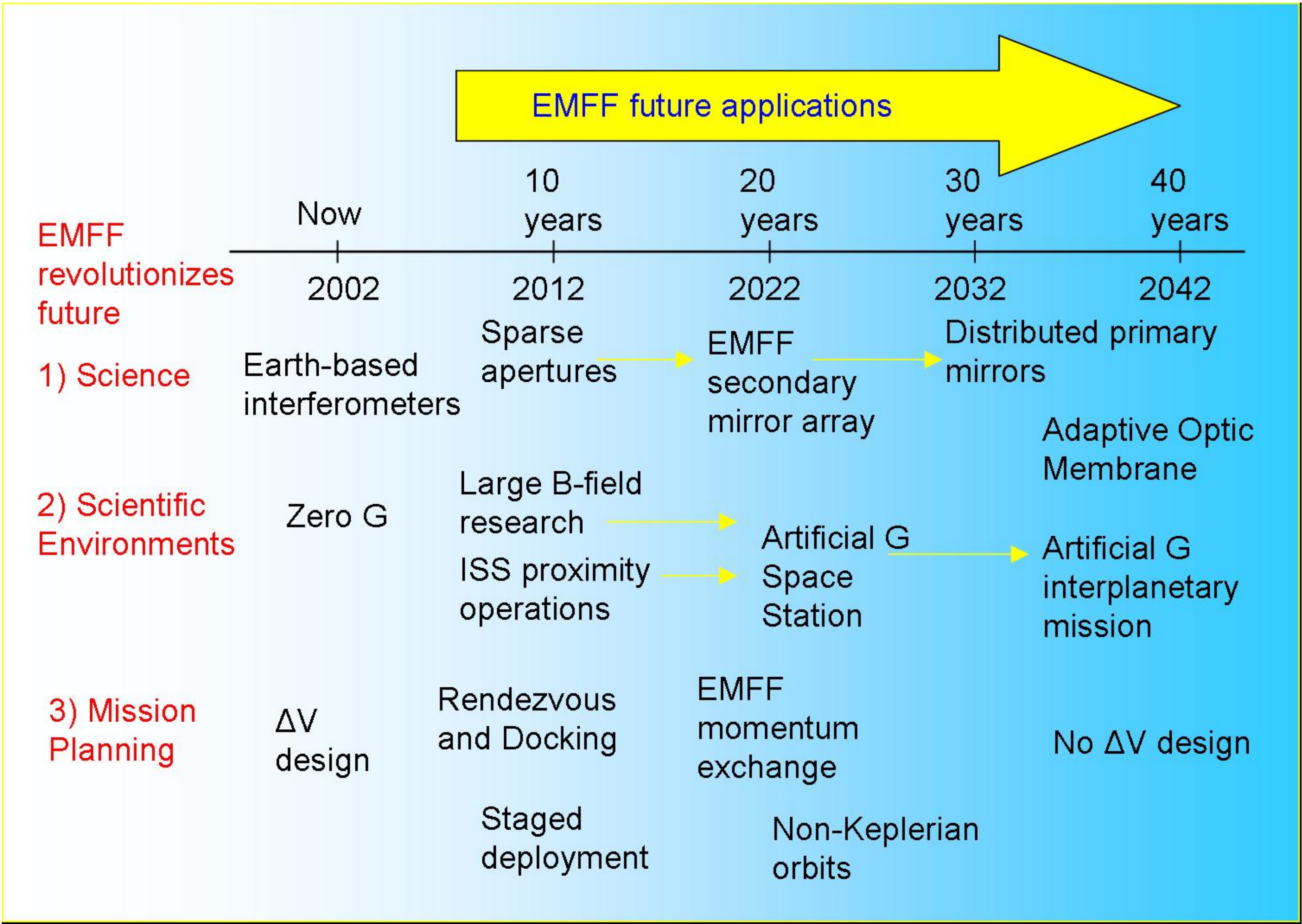
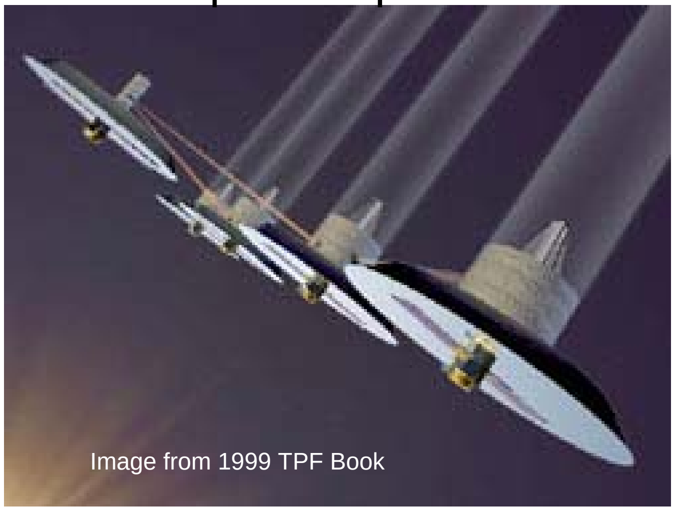
- **MIT EMFFORCE Testbed**
  - Design
  - Calibration
  - Movie
- **Space Hardware Design Issues**
  - Thermal Control
  - Power System Design
  - High B-Field Effects
- **Conclusions**

EMFF Roadmap

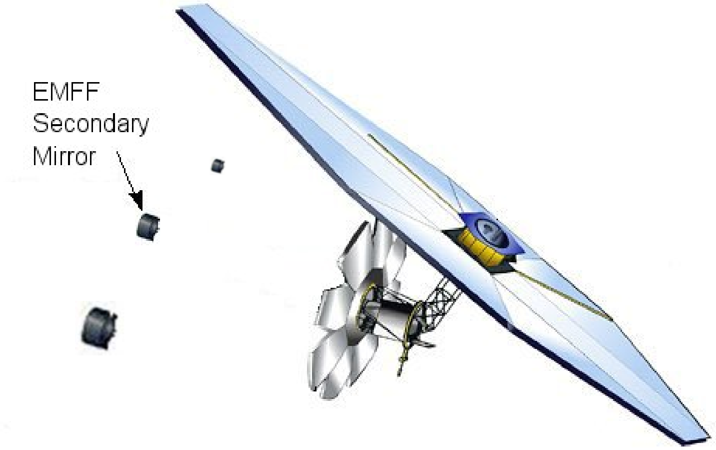


EMFF Applications in 10-20 Years

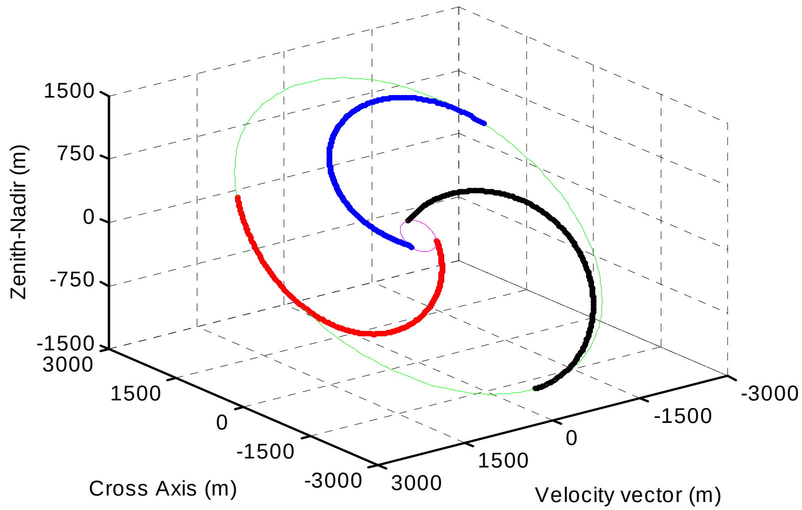
Sparse Apertures



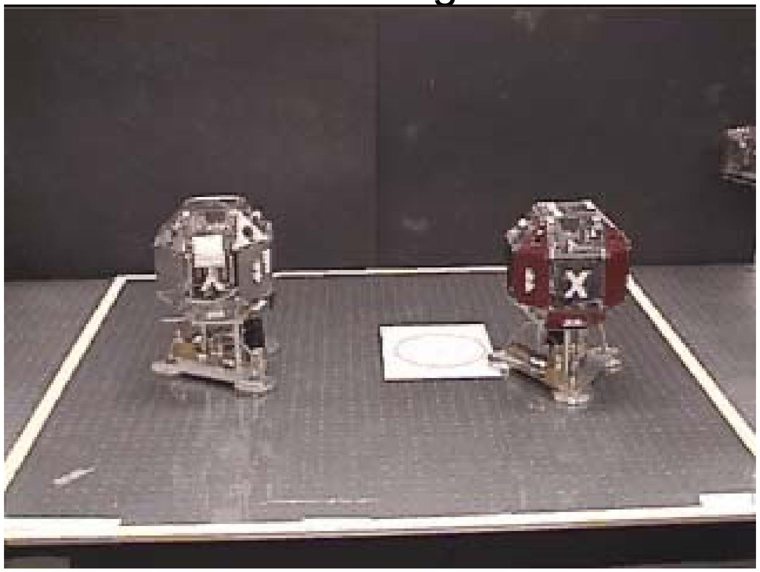
EMFF Secondary Mirrors



Cluster Reconfiguring

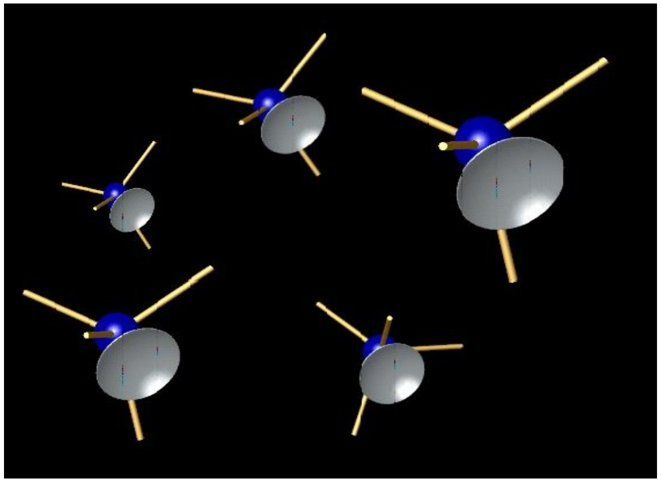


Docking

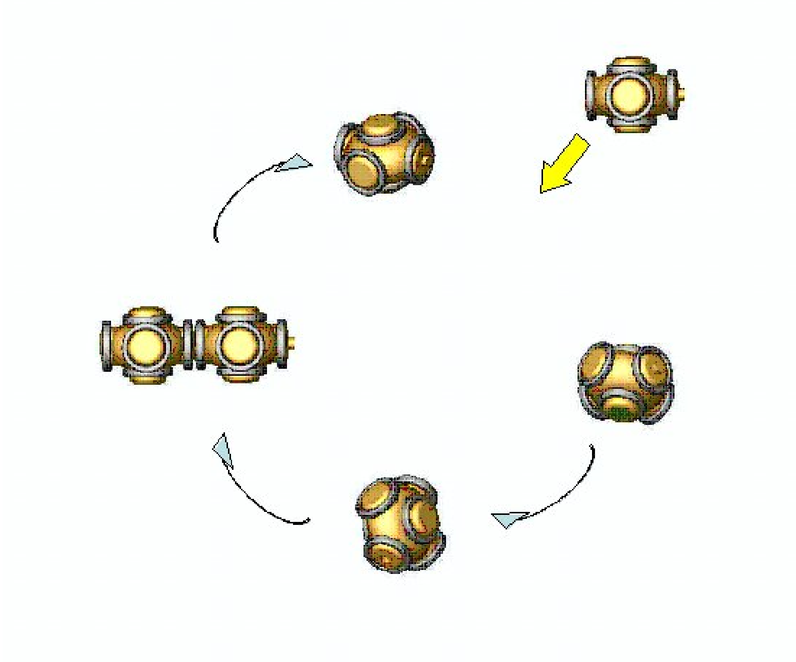


EMFF Applications in 30-40 Years

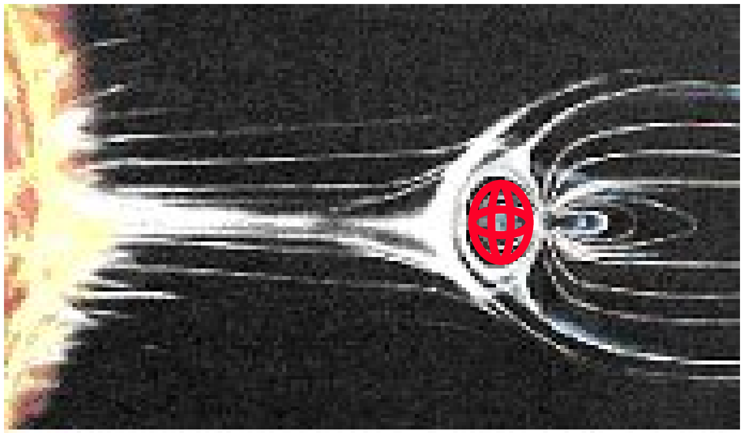
Reconfigurable Arrays & Staged Deployment



Reconfigurable Artificial Gravity
Space Station

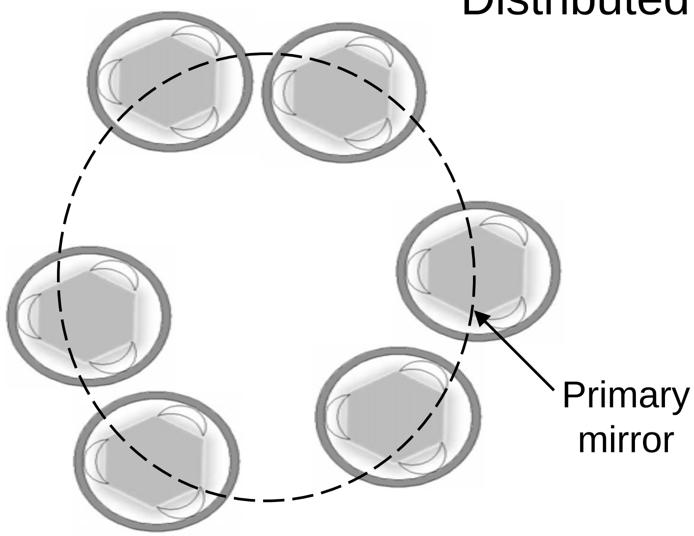


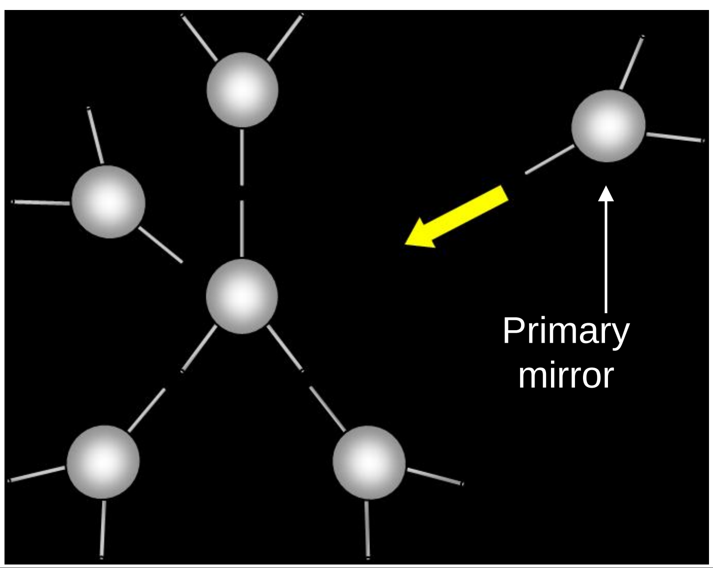
Protective magnetosphere



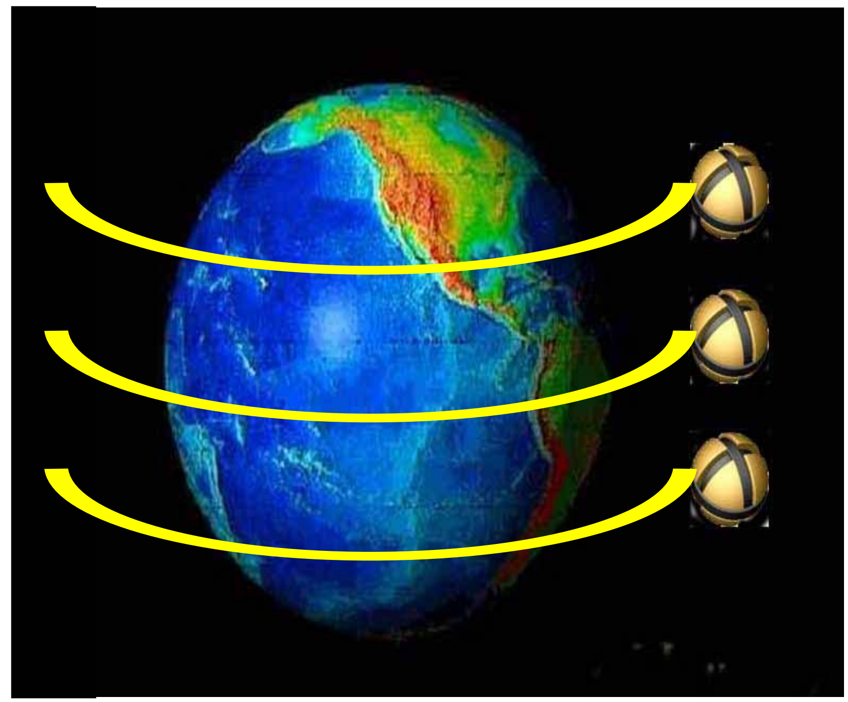
Additional Mission Applications

Distributed Optics





Non-Keplerian Orbits



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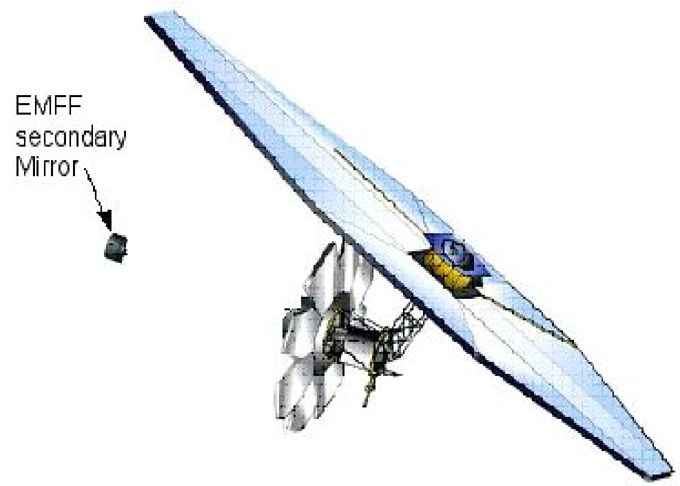
‘Stationary’ Orbits

- For telescopes and other observation missions with an extended look time, holding an fixed observation angle is important
- Satellite formations in Earth’s orbit have an intrinsic rotation rate of 1 rev/orbit
- EMFF can be used to stop this rotation and provide a steady Earth relative angle.
- Using Hill’s equations…



$$\ddot{x} = 3n^2 x + 2n\dot{y} + a_x$$

$$\ddot{y} = -2n\dot{x} + a_y$$

$$\ddot{z} = -n^2 z + a_z$$

$$\vec{f} = -3x\,m\,n^2\ \hat{x} + z\,m\,n^2\hat{z}$$

$$\vec{\tau} = \vec{x} \times \vec{f} \qquad \vec{\tau} = m\,n^2 \begin{pmatrix} y\,z \\ -4x\,z \\ 3x\,y \end{pmatrix}$$

- Unless the required force vector aligns with the position vector, torques are produced
  - Zero torque solutions are
    - Holding a satellite in the nadir direction
    - Holding a satellite in the cross-track direction
- For other pointing angles, torques will be produced
- Any angular momentum buildup can be removed by:
  - Moving to an opposite position.
  - Interacting with the Earth’s magnetic field

Rotating Linear Array: 2 vs. 3 Spacecraft

Two Spacecraft

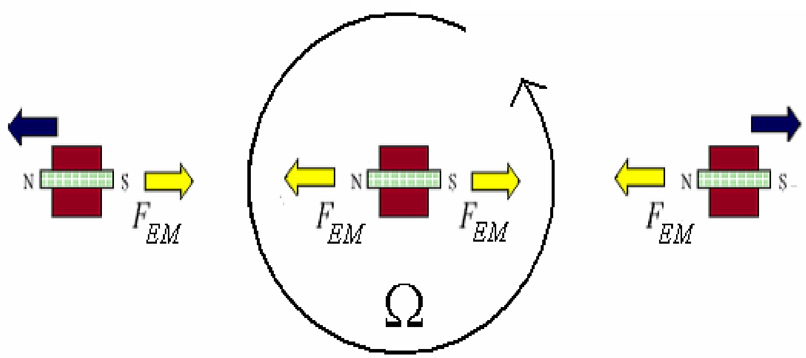
$$F = \frac{3}{2}\mu_o \pi \frac{n^2 i^2 a^4}{B^4} = \frac{1}{2} m_{tot}\omega^2 B$$

$$m_{array} = 2m_{tot}$$

Three Spacecraft

$$F = \frac{3}{2}\mu_o \pi a^4 \left(\frac{n_i i_i n_o i_o}{(\frac{B}{2})^4} + \frac{n_o^2 i_o^2}{B^4} \right) = \frac{1}{2} m_{tot}\omega^2 B$$

$$m_{array} = 2m_{tot_{outer}} + m_{tot_{inner}}$$



Mission Efficiency metric:

$$J = \frac{w}{c_o m_{array}} \qquad c_o = \sqrt{\frac{3\mu_o \pi}{B^5}}$$

$$\frac{J_3}{J_2} = 2.75$$

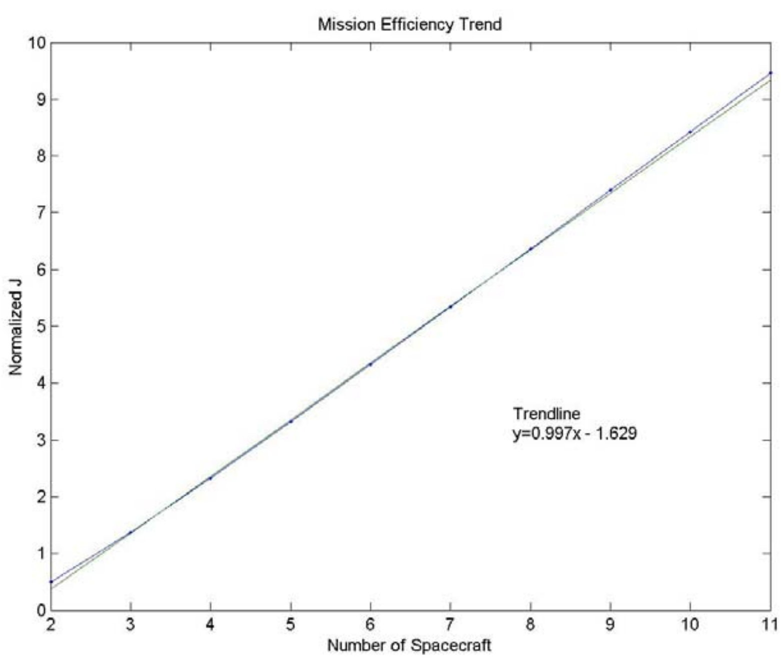
- Adding combiner almost triples mission efficiency
- Trend Continues → adding more spacecraft increases mission efficiency

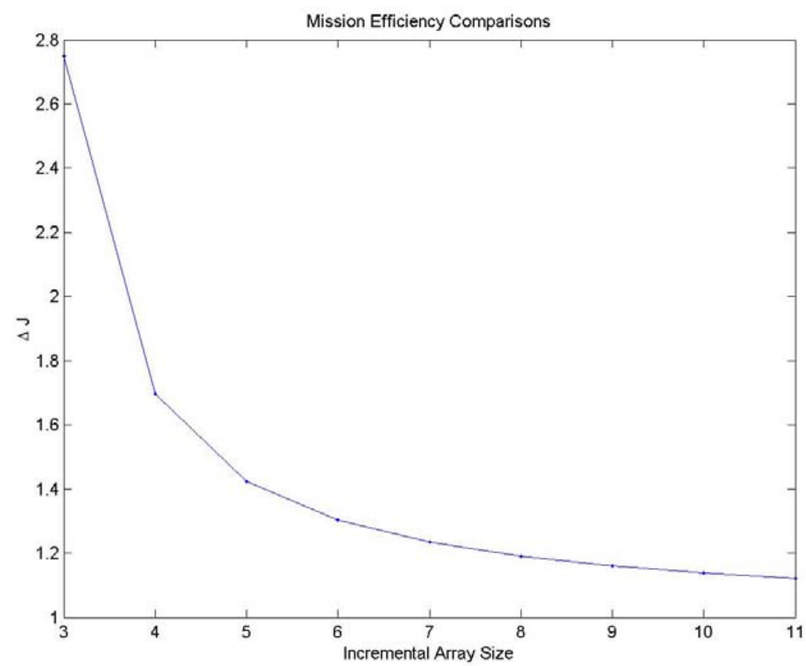
RLA: Mission Efficiency Trends

- Normalized Mission Efficiency

$$J_0 = \frac{nia^2}{c_o m_{array}} \Rightarrow J_n = \frac{\sqrt{\sum_{i=1}^{n-1}\left(\frac{i}{n-1}\right)^{-4}}}{n}$$

- Comparing J_3/J_2, then J_4/J_3, J_5/J_4, J_6/J_5, etc.
- Diminishing returns of adding S/C





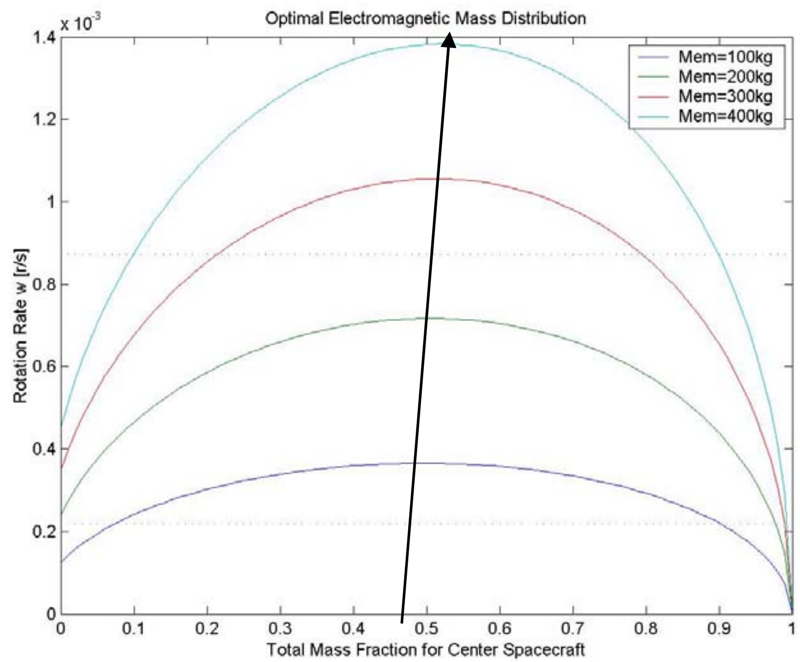
3 S/C RLA: EMFF System Trades

•Identical or Mother-Daughter Configuration

•Define Mass Fractions:

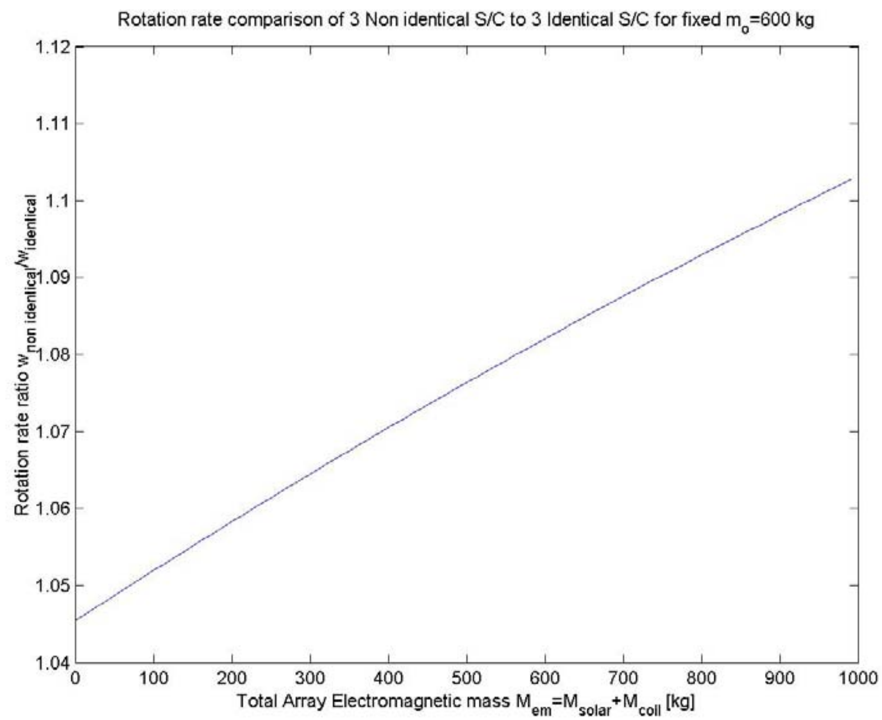
$$M_{inner} = \gamma M_{total_{array}}$$

$$M_{outer} = \frac{\gamma - 1}{2} M_{total_{array}}$$



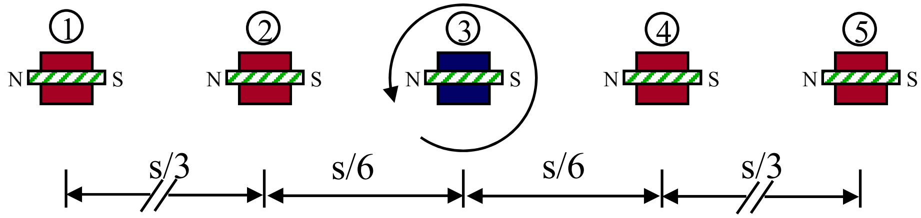
•Identical Configuration is non-optimal

Center Spacecraft experiences no translation → no mass penalty → suggests larger center spacecraft



•Higher rotation rate for mother-daughter configuration for fixed masses

Case Study: Sparse aperture (TPF)



- Compare total system mass for various propulsion options with EM option for the TPF mission (4 collector and 1 combiner spacecraft)
- Array is to rotate at a fixed rotation rate (ω = 1rev/2 hours)
- All collector spacecraft have same EM core and coil design
- All spacecraft have the same core
- Force balancing equations:

$$F_{cent_1} = F_{M_{12}} + F_{M_{13}} + F_{M_{14}} + F_{M_{15}}$$
$$F_{cent_2} = -F_{M_{21}} + F_{M_{23}} + F_{M_{24}} + F_{M_{25}}$$

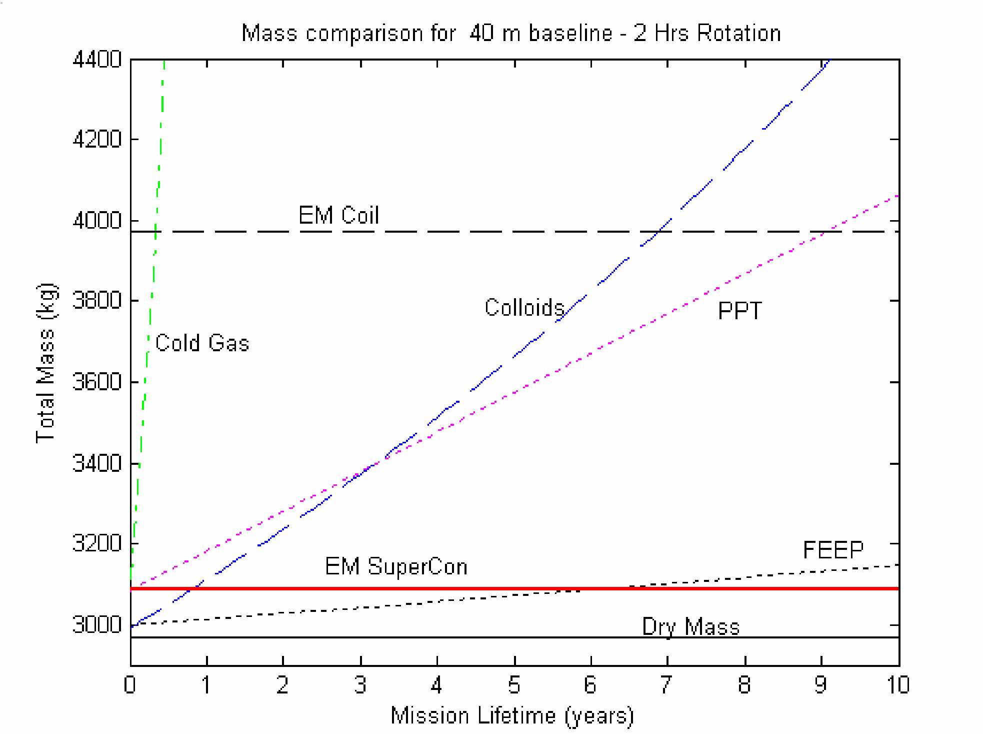
EM mass components

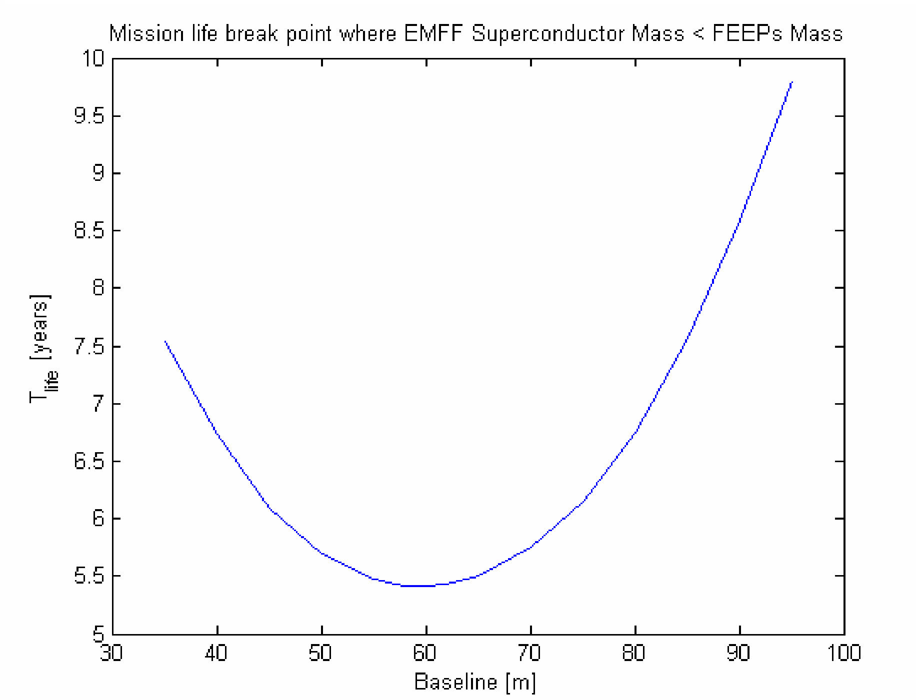
$$m_{sc} = m_{dry} + m_{sa} + m_{core} + m_{coil}$$

TPF spacecraft* (m_{dry})	
Collector Spacecraft	
Dry	600 kg, 268 W
Propulsion	96 kg, 300 W
Propellant	35 kg
Combiner Spacecraft	
Dry	568 kg, 687 W
Propulsion	96 kg, 300 W
Propellant	23 kg
Solar Array (m_{sa})	
Power to mass conv (P_{conv})	25 W kg^{-1}
Superconducting wire (m_{sc})	
Density (ρ_{St})	13608 kg m^{-3}
Copper coil (m_{coil})	
Density (ρ_{Cu})	8950 kg m^{-3}
Resistivity (ρ)	1.7x10^{-8} Ωm

*Source: TPF Book (JPL 99-3)

Case Study: Sparse aperture (TPF-2)





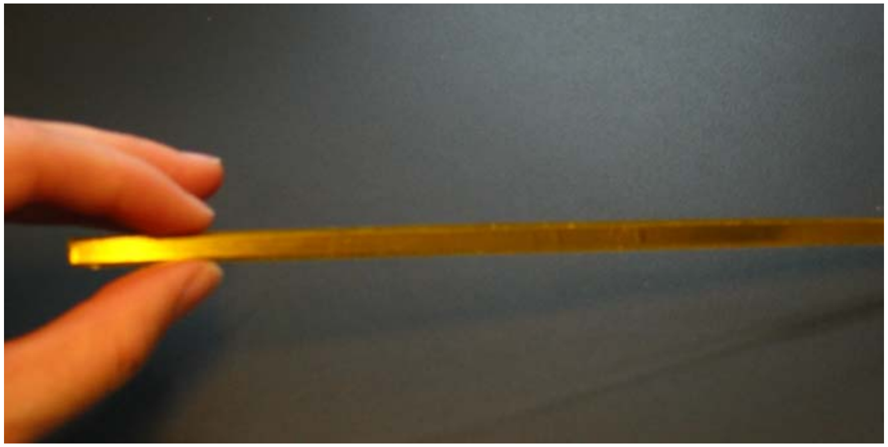
- ***Cold Gas -*** Low I_{sp}, high propellant requirements
  - Not viable option
- ***PPTs and Colloids -*** Higher I_{sp}
  - still significant propellant over mission lifetime
- ***FEEPs –*** Best for 5 yr mission lifetime
  - Must consider contamination issue
  - Only 15 kg mass savings over EMFF @ 5 yr mark
- ***EM coil*** (R = 4 m) (M_{tot} = 3971 kg)
  - Less ideal option when compared to FEEPs even for long mission lifetime
- ***EM Super Conducting Coil*** (R = 2 m) (M_{tot} = 3050 kg)
  - Best mass option for missions > 6.8 years
  - No additional mass to increase mission lifetime
  - Additional mass may be necessary for CG offset
    - Estimated as ~80 kg

EMFF Testbed

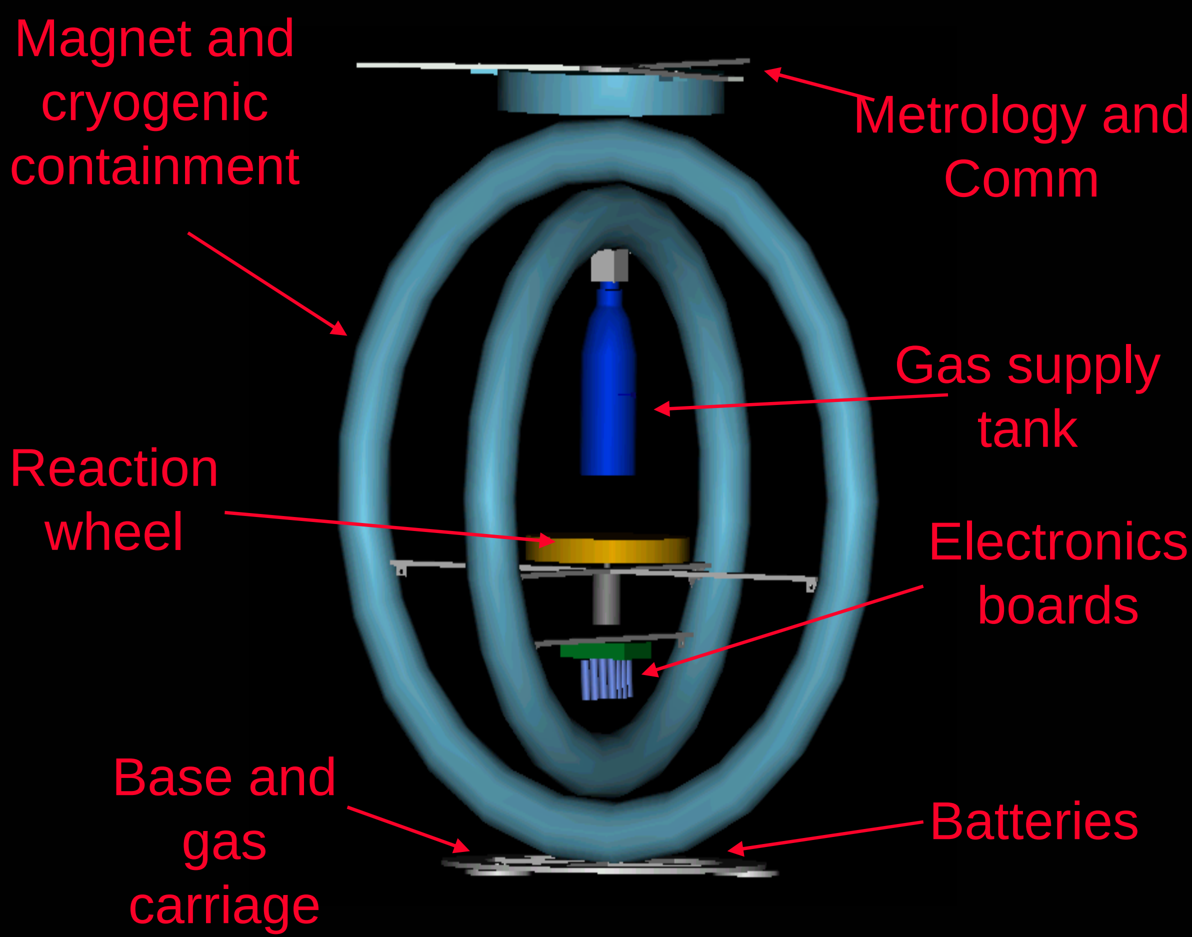
EMFFORCE Project Overview

- Goal: Demonstrate the feasibility of electromagnetic control for formation flying satellites

- Design and build a testbed to demonstrate 2-D formation flight with EM control
  - Proof of concept
  - Traceable to 3-D
  - Validate enabling technologies
    - High temperature superconducting wire



From Design to Reality



Testbed Overview

- Functional Requirements:
  - System will contain 2 vehicles
  - Robust electromagnetic control will replace thrusters
  - Each vehicle will be:
    - Self-contained (no umbilicals)
    - Identical/interchangeable

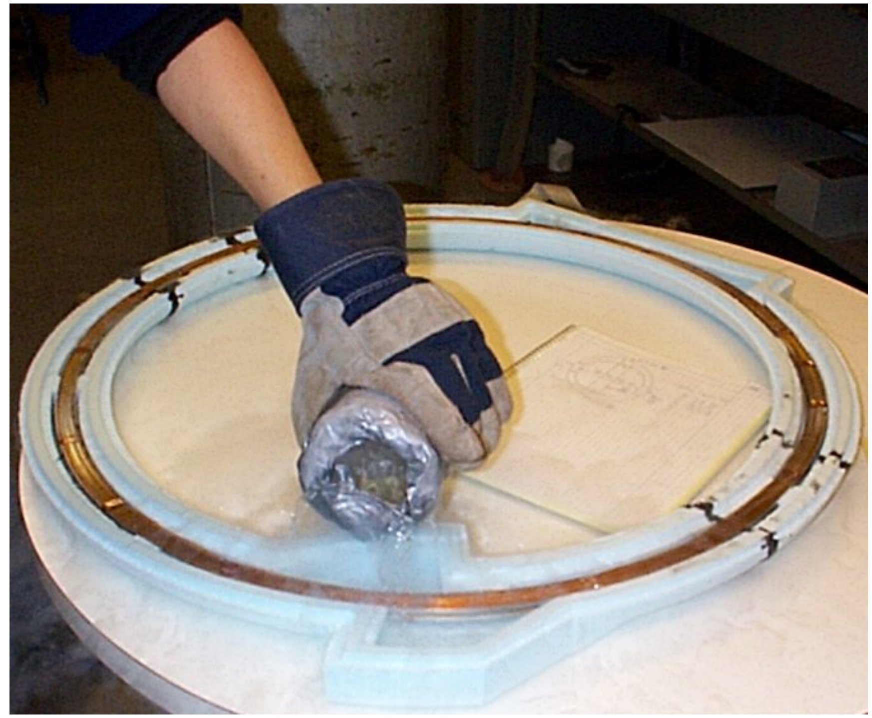
- Vehicle Characteristics
  - Each with 19 kg mass, 2 electromagnets, 1 reaction wheel

- Communication and processing
  - 2 internal microprocessors (metrology, avionics/control)
  - Inter-vehicle communication via RF channel
  - External "ground station" computer (operations, records)

- Metrology per vehicle
  - 1 rate gyro to supply angular rate about vertical axis
  - 3 ultrasonic (US) receivers synchronized using infrared (IR) pulses

Electromagnet Design

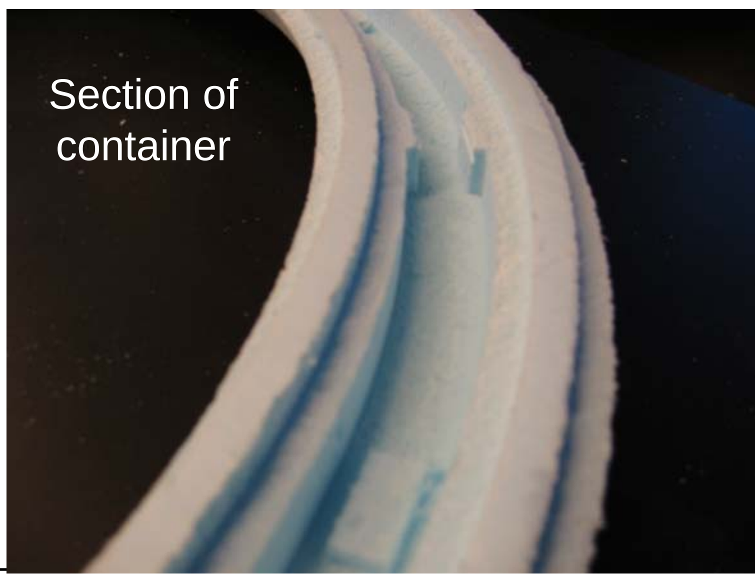
- American Superconductor Bi-2223 Reinforced High Temperature Superconductor Wire
  - Dimensions
    - 4.1 mm wide
    - 0.3 mm thick
    - 85 m length pieces
  - Critical Current
    - 115 amps, 9.2 kA/cm2
      - Below 110 K

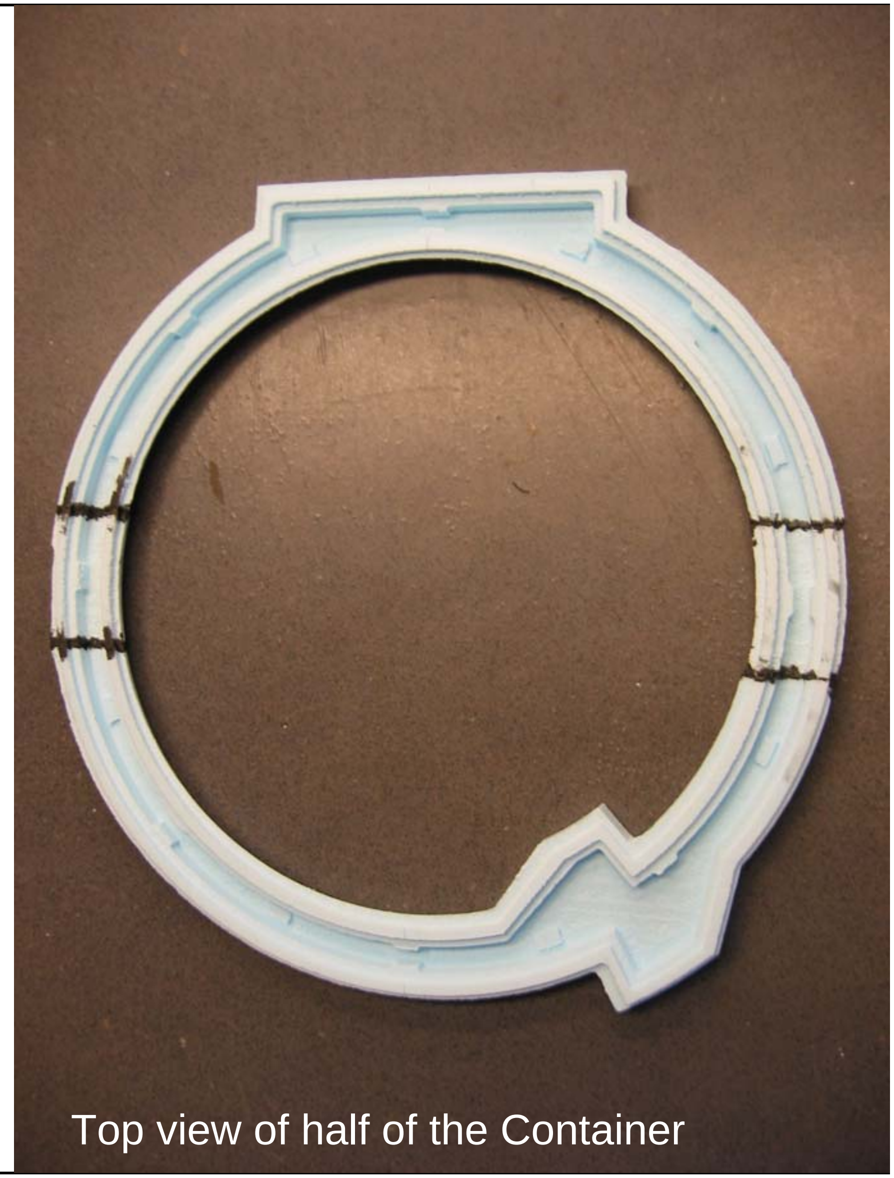


- Coil wrapped with alternating layers of wire and Kapton insulation
  - 100 wraps
  - Radii of 0.375m and 0.345m
- Toroid-shaped casing: Insulation & Structural component
    - Operable temperature at 77 K
    - Surround by liquid nitrogen

Containment System Design

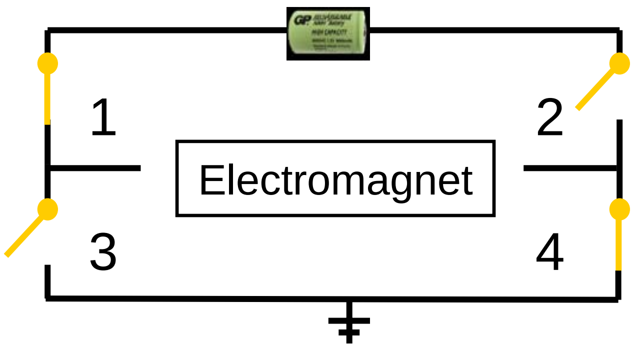
- Requirements:
  - Keep the wire immersed in liquid nitrogen.
  - Insulate from the environment the wire and the liquid N_2.
    - Non-conductive material.
  - Stiff enough to support liquid N_2 container and its own weight.
- Material: Foam with fiber glass wrapped around it.

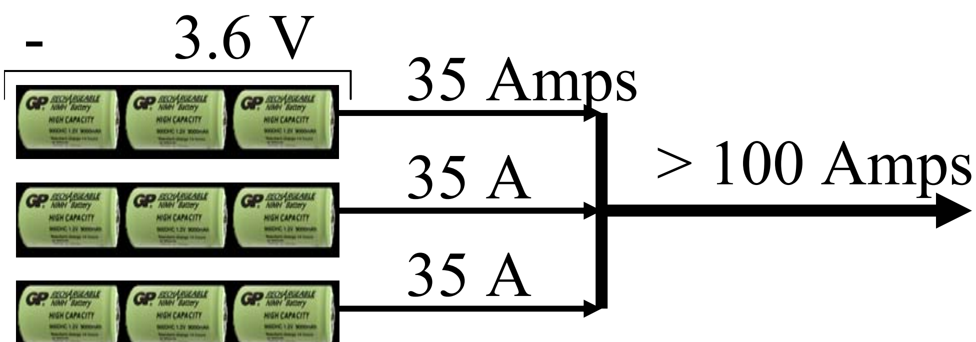

Section of container


Top view of half of the Container

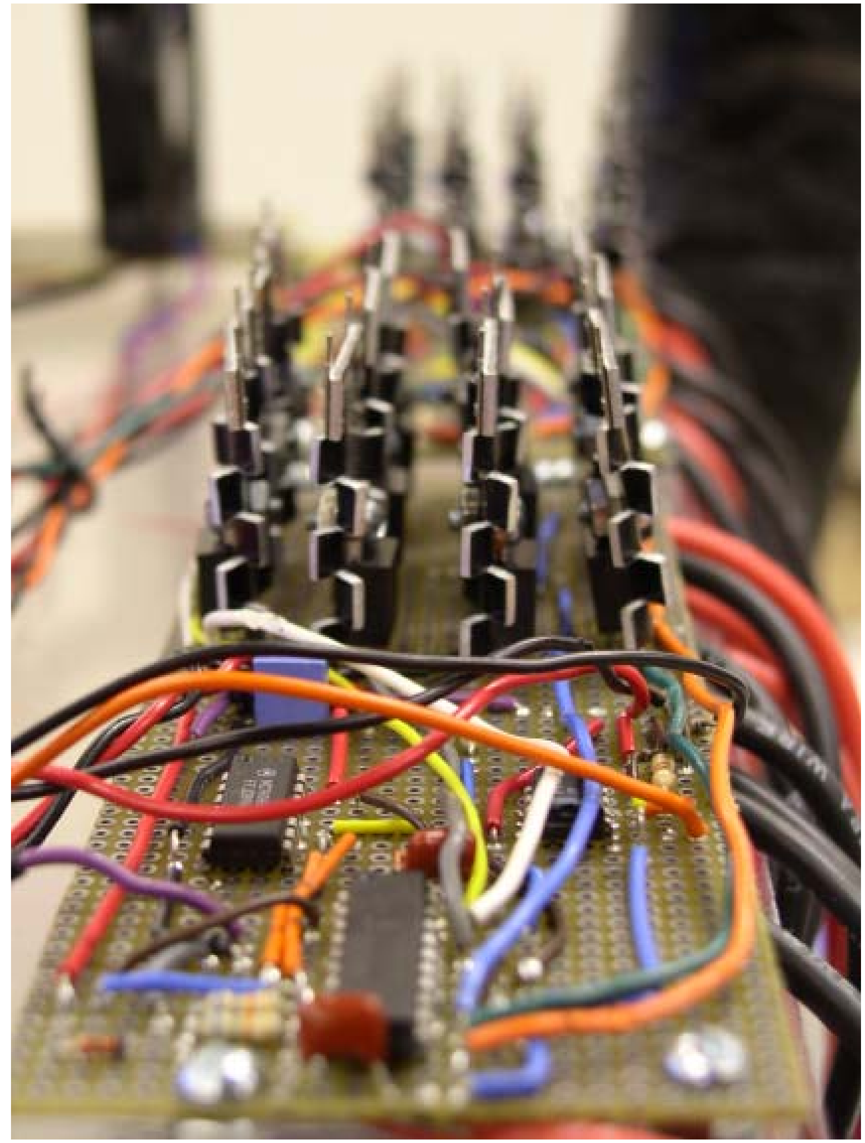
Power Subsystem

- Coil & Reaction Wheel Power:
  - Rechargeable NiMH D-cell batteries
  - MOSFET controller – uses H-bridge circuit to control current through gates
  - 20 minute power duration

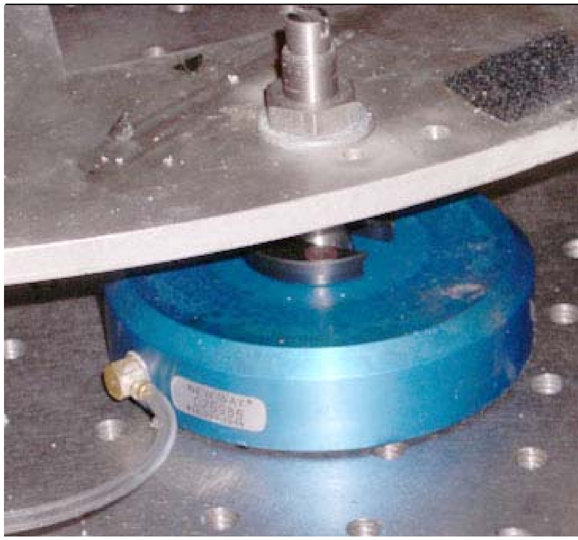




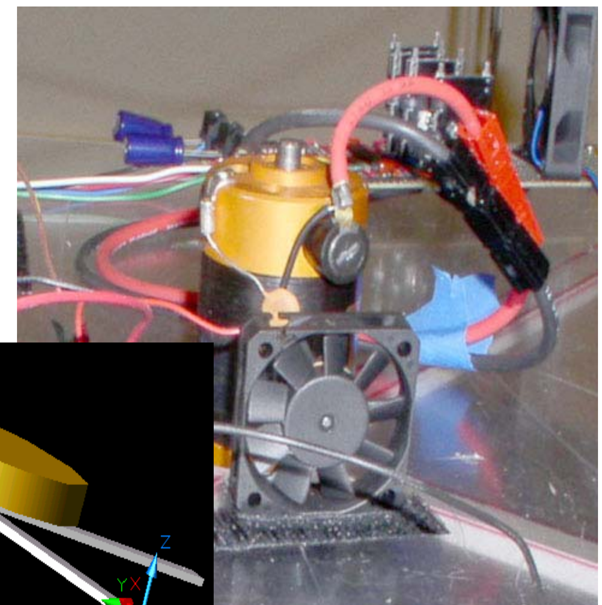
- Coil: 100 Amps @ 5 Volts



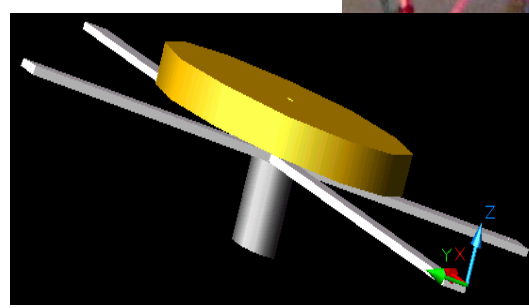
Air Carriage and Reaction Wheel



- 2-D Friction-less environment provided by gas carriage
  - allows demonstration of shear forces, in concert with reaction wheel
  - Porous Membrane, Flat air bearings provide pressurized cushion of gas
  - CO_2 gas supply: rechargeable compressed gas tank, 20 minute duration

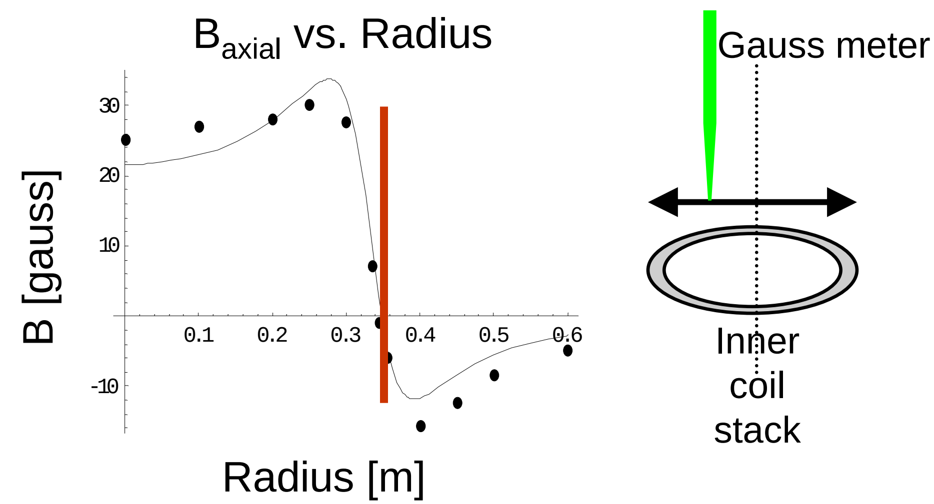


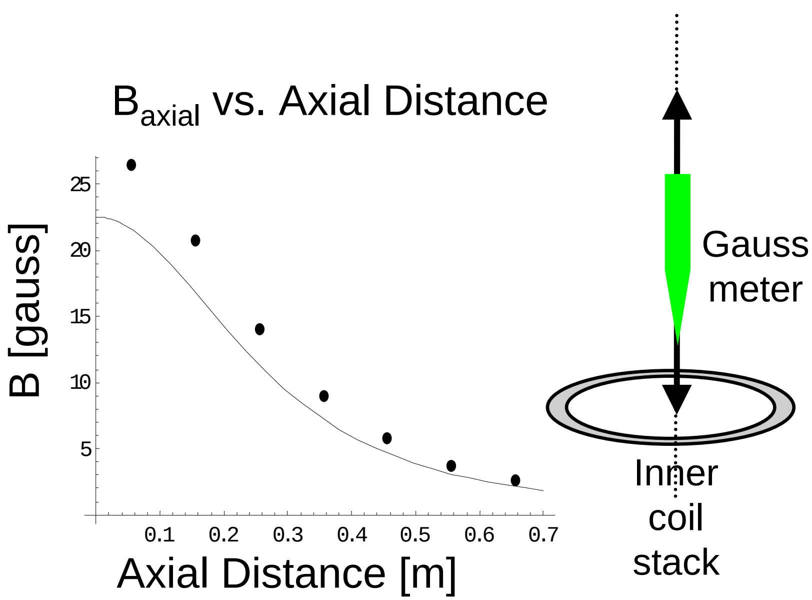
- Reaction Wheel
- Store angular momentum
  - Provide counter-torques to electromagnets
  - Provide angular control authority
  - 0.1 Nm Torque at 10 Amps
- Flywheel Requirements:
  - non-metallic → Urethane Fly Wheel
  - Maximum wheel velocity at 7000 RPM
- Motor tested in EM field with no variation in performance

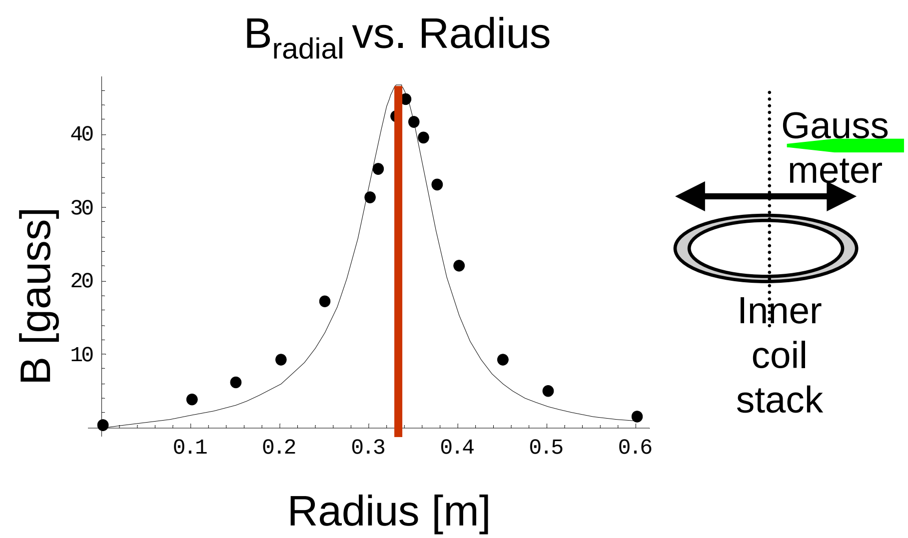


Model Calibration

- B-field measurements
  - Axial and Radial B-field measurements taken at varying radii.
  - Inner coil stack
    - .67 m inner diameter
    - 40 turns
    - I = 30 amps

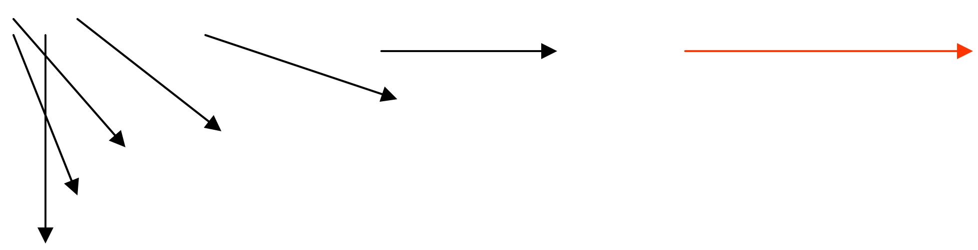


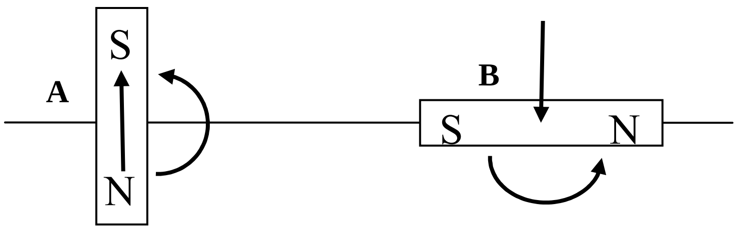




Degrees of Freedom Validation

- Initially we had problems demonstrating shear forces
- The reaction wheel is designed for small shear forces
- Vehicle tends to 'stick' to table, so larger forces are needed to move the vehicle
- Larger shear forces produce larger torques
- The torque generated would cause the vehicle to rotate
- As the vehicle rotated, the dipoles aligned causing the vehicles to attract
- Used Vehicle's ability to steer the dipole to compensate





EMFF: Validation of Degrees of Freedom

Testbed Future Work

- Control Testing
  a. One vehicle fixed – disturbance rejection
  b. One vehicle fixed – slewing, trajectory following
  c. Both vehicles free – disturbance rejection
  d. Both vehicles free – slewing, trajectory following
  e. Spin-up

- **Vehicle Design**
  - Containment system redesign: Plastic or copper tubing
  - Reaction Wheel
    - Motor is too weak to counteract high torque levels
    - Reaction wheel is also possibly undersized

- **Three vehicle Control Testing**

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Cryogenic Containment

- Significant research concerning maintaining cryogenic temperatures in space
  - Space Telescope Instrumentation
  - Cryogenic propellant storage
- Spacecraft out of Earth orbit can use a sunshield that is always sun-pointing to reflect radiant energy away
- For Earth orbit operation, this won't work, since even Earth albedo will heat the 'cold' side of the spacecraft
- A cryogenic containment system, similar in concept to that used for the EMFF testbed must be implemented, using a combination of a reflective outer coating, good insulation, and a cyo-cooler to extract heat from the coil
- Using a working fluid to carry heat around to the cry-cooler will be explored, or possibly using the wire itself as the thermal conductor

Efficient High Current Supplies

- The existing controller for the testbed was based on a pulse width modulated controller found for use with radio controlled cars and planes

- An H-bridge is used to alternate applied potential to the coil, with the next current delivered dependent on the amount of time the voltage is applied in a given direction

- The drawback is that current is always flowing through the batteries, which both provide a power sink as well as dissipate heat

- One solution may be to incorporate very high Farad capacitor instead of a batter, to reduce the internal resistance

- Alternatively, a method of 'side-stepping' the storage device altogether may be employed, allowing the current to free-wheel during periods of low fluctuation

High B-field effects: Findings in literature

- NASA reports, Lockheed Martin reports, other contractors (when available), IEEE journal articles

- Nothing for very high fields (0.1 T and above)

- Effects of earth's magnetic field (0.3 gauss or so)

- Effects of on-board field sources such as
  - Magnetic latching relays
  - Traveling wave tubes
  - Tape recorders
  - Coaxial switches
  - Transformers
  - Solenoid valves
  - Motors

Vulnerable equipment

- All these fields are much smaller than what is being projected for magnetic steering coils
- Equipment traditionally known to be susceptible to magnetic effects:
  - Magnetometers
  - Photomultipliers
  - Image-dissector tubes
  - Magnetic memories
  - Low-energy particle detectors
  - Tape recorders
- Digicon detectors in Hubble FOS were found to be vulnerable to magnetic effects
- Quartz-crystal oscillators ditto (AC fields)

High field concerns

- Other effects may come into play that are negligible at low field strengths
  - Eddy currents in metal harnesses
  - Hall effects in conductors
  - Effects in semiconductors?
- Most EMI requirements hard to meet
- Shielding requirement translates into a mass penalty
- Pursuing more literature results, but this is effectively a new regime – may require testing

Shielding Considerations

- Attenuation of a DC magnetic field resulting from an enclosure scales approximately as

$$A = \frac{\mu}{2}\frac{\Delta}{R}$$

- Where μ is the permeability, Δ is the thickness of the material, and R is the characteristic radius of enclosure
- Some high permeability materials:

Material	Density (lbs/cu-in)	Permeability	Saturation (G)
Amumetal	0.316	400000	8000
Amunickel	0.294	150000	15000
ULCS	0.0283	4000	22000

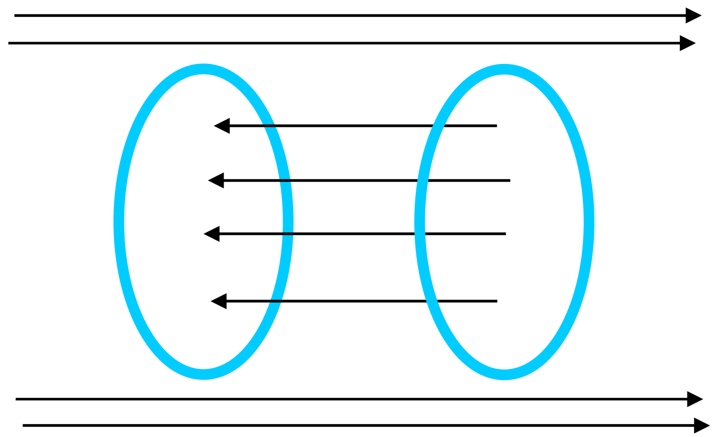
- Reducing a 600 G (0.06 T) field to ambient (0.3 G) requires an attenuation of 2×10^3, or a minimum Δ/R of 0.01
- This is .1 mm thickness for each 10 cm of radius enclosed

Further Shielding Considerations

- Geometry
  - Shielding acts to divert field lines around components
  - Gentle radii are better for re-directing field lines than sharp corners
- Size
  - Smaller radii are more effective, so shielding should envelop the component to be protected as closely as possible
- Continuity
  - Separate pieces should be effectively connected either mechanically or by welding to insure low reluctance
- Closure
  - Components should be completely enclosed, even if by a rectangular box to shield all axes
- Openings
  - As a rule, fields can extend through a hole ~5x the diameter of the hole
- Nested Shields
  - In high field areas, multiple shield layers with air gaps can be used very effectively. Lower permeability, higher saturation materials should be used closer to the high field regions

Shielding with Auxiliary Coils

- In addition to high permeability materials, shielding can be achieved locally using Helmholtz coils



- An external field can be nullified with an arrangement of coils close to the region of interest
- The small coil size requires proportionally smaller amp-turns to achieve nulling of the field
  - Will not significantly affect the main field externally

Outline

- **Motivation**
- **Fundamental Principles**
  - Governing Equations
  - Trajectory Mechanics
  - Stability and Control
- **Mission Applicability**
  - Sparse Arrays
  - Filled Apertures
  - Other Proximity Operations
- **Mission Analyses**
  - Sparse Arrays
  - Filled Apertures
  - Other Proximity Operations
- **MIT EMFFORCE Testbed**
  - Design
  - Calibration
  - Movie
- **Space Hardware Design Issues**
  - Thermal Control
  - Power System Design
  - High B-Field Effects
- **Conclusions**

Conclusions

- There are many types of missions that can benefit from propellantless relative control between satellites
  - Provides longer lifetime (even for aggressive maneuvers)
  - Reduces contamination and degradation
- Angular momentum management is an important issue, and methods are being developed to de-saturate the reaction wheels without using thrusters
- Preliminary experimental results indicate that we are able to perform disturbance rejection in steady state spin dynamics for multiple satellites
- Optimal system sizing has been determined for relatively small satellite arrays. Currently larger formations are being investigated
- Although low frequency magnetic interference data is difficult to find, shielding against the relatively low fields inside the coils appears to be possible
- Preliminary validation with the MIT Testbed has been achieved, and more complex maneuver profiles will be accomplished with future upgrades