Massachusetts Institute of Technology
Department of Aeronautics and Astronautics

16.682 – Special Subjects in Aeronautics and Astronautics
"Prototyping Avionics"

<u>Homework #1</u>
Out: Mon Feb 13, 2006
Due: **Wed** Feb 22, 2006

Topics:
- Basic laws
- Discrete Components
- Operational Amplifiers (Op-Amps)

Problem 1 – Basic Laws

In class we reviewed four basic laws for circuits. These laws results in the four most important equations you will use when working with both analog and even digital circuits, therefore it is important that you have good intuition on what they mean.

1) State the four component laws (tip: they relate current, voltage, resistance, power, capacitance, and inductance).

2) For each of the four laws, provide an explanation of how you would explain these laws to make them intuitive (please use your own analogies if you have any different from the ones we used in class):
   a) How would you interpret resistor networks intuitively?
   b) How do capacitors behave under constant voltage and during transitions?
   c) How do inductors behave under constant current and during transitions?

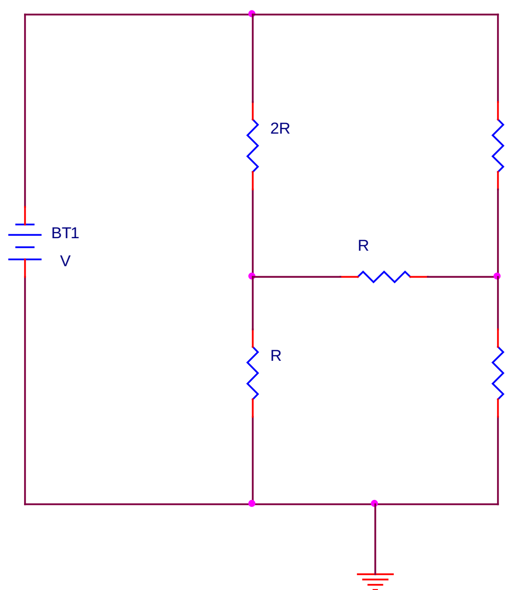
3) During lecture it was stated multiple times that these laws are for "ideal" components. Describe using words and/or diagrams what are the non-ideal features of a real resistor, capacitor, and inductor.

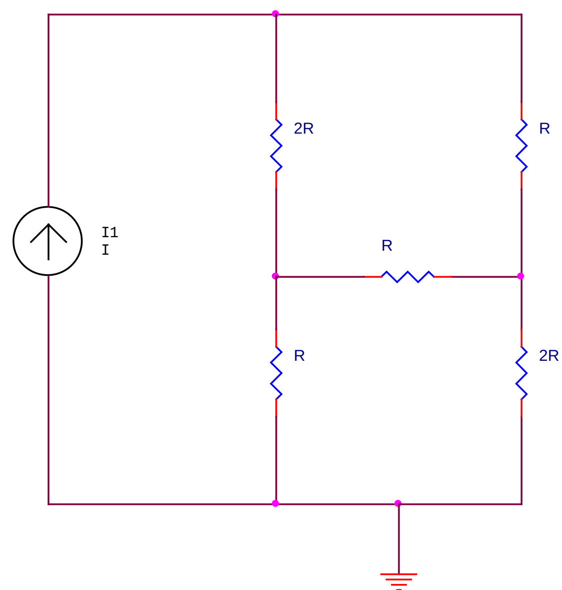
4) These four laws require that you know two other laws to determine how a circuit behaves: Kirchoff's current (KCL) and voltage (KVL) laws. State these two laws and explain them using both words and diagrams.

Problem 2 – Discrete Components

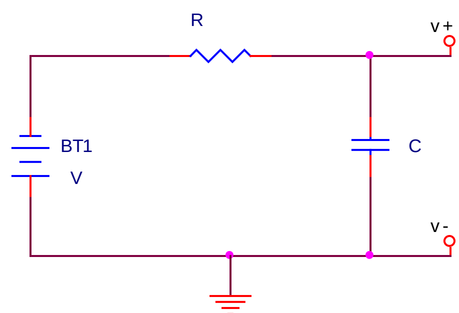
1) Resistor Networks – The following resistor networks check your understanding of KCL and KVL.
   a) Normally the first step with a resistor network is to figure out if you can reduce the network to an equivalent circuit using the rules for series and parallel resistors. Can you reduce this network? Why or why not?
   b) Find the voltages at all the nodes (tip: 4 total nodes, 2 are interesting) and the current through all resistors when the battery "BT1" provides constant voltage V.
   c) Find the voltages at all the nodes and the current through all resistors (5) when the component "I1" provides constant current.

   Express all your answers in terms of V, I, and R.

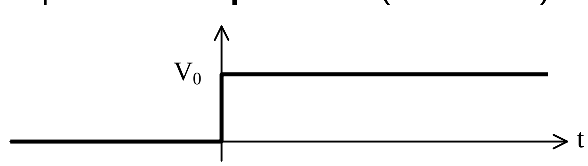




2) Capacitors – A major role of capacitors in electronics is to help filter signals. Lets examine the circuit below, following what we learned in class (and P1.2), and then confirm it with differential equations



The input V is a **step** function (as in class):



a) What is the voltage across the capacitor at time $t=0^+$?
(t=0^+ means immediately after the step.)
(Tip: what is the slope of input voltage V and what does it mean given that V=IR?)

b) What is the voltage across the capacitor at time $t=\infty$?

c) Intuitively from the equations in class, what is the shape of the voltage across the capacitor?

d) Now lets solve the differential equation to find the voltage across C and confirm our intuition:

KVL gives you the following relation, use it to derive the differential equation and solve it:

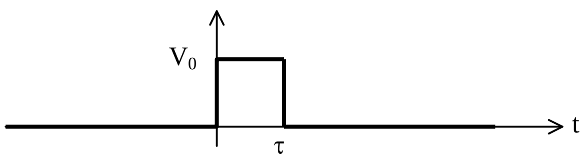
Battery V = Voltage across R + Voltage across C

KCL tells you that:

Current through R = Current through C

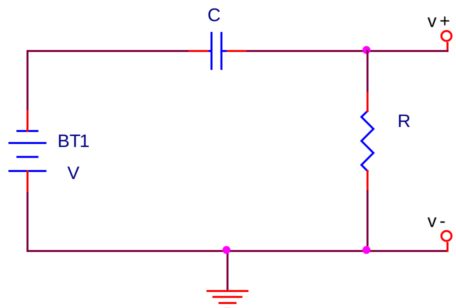
Use both of these to get the equation; a quick review of 8.03 should help you solve it.
You should see a solution very similar to what we saw in class.

e) Now assume the voltage is a **step**:

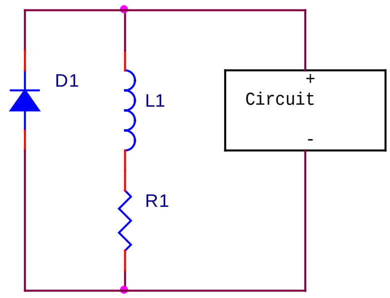


What value of C (any) and R (between 1kΩ and 10MΩ) would you choose so that a noise pulse of τ=10ms is filtered to less than 10% of V_0?

f) Without necessarily deriving all the equations (you can do it if its easier), what does the following circuit do? Explain. (Tip: it is also a filter.)



3) Inductors and Diodes – While we will not use inductors very often, they exist in the real world and can give you headaches. This problem deals with what is normally an "advanced" concept to prevent circuit failures, since it actually depends on very simple concepts.
Consider the following circuit:



The box labeled "Circuit" is a **power source** which behaves as follows:

- Before time t=0 it maintains a positive voltage between its +/- leads of V_0.
- At time t=0 it "opens" the circuit and no longer "drives" inductor L1.
- It is required that voltage is not applied externally to this circuit.
- While the circuit behaves almost as an open circuit when it is off, it **does** have some finite resistance through it, which means a small amount of current will flow through it.

Lets examine what is going on:

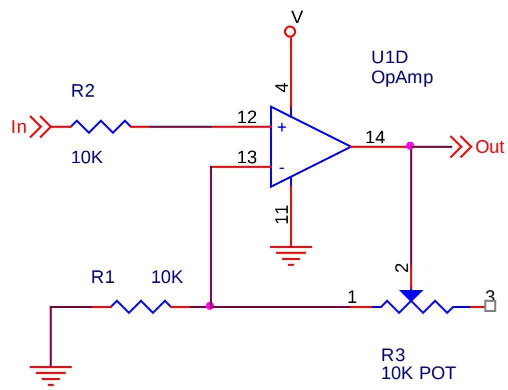
a) Ignore the diode completely, and examine the Zero State Response of the Inductor:
   i) From P1.2, and without the diode, how does the circuit look at $t=0^+$? (Draw the "circuit" box as a resistor.)
   ii) Since the inductor is an energy storage and it has a resistive load on it (the circuit box which is off), it is acting as a power-supply! How does the current flow?
   iii) What do you expect the voltage to be across the inductor, assuming the circuit box has an equivalent "off resistance" of R0 (large, but finite)?

b) Now lets look at the diode… what does it do?
   (Tip: Remember that the diode has practically zero resistance when current goes through it the right way.)

Problem 3 – Operational Amplifiers

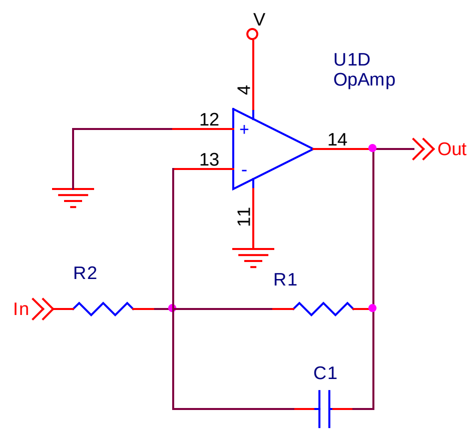
In this problem we will examine several operational amplifiers that are very helpful when trying to capture analog signals.

1) Examine the following circuit:



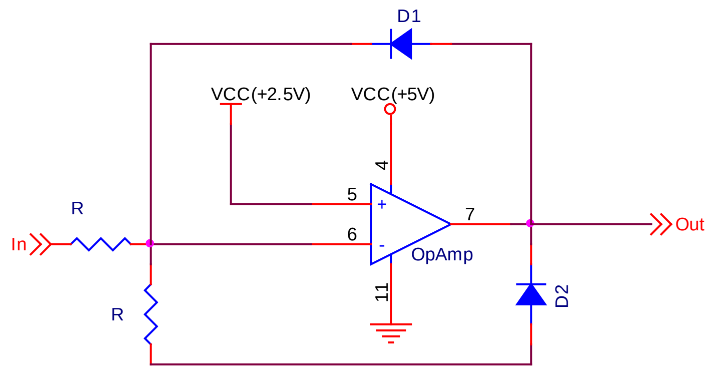
a) What are the maximum/minimum output voltages?
b) Without solving any equations, and using only the fact that $v^+ \approx v^-$, can you determine what V_{out} is when the potentiometer (R3) is set to 0Ω (a short)? If so, what is it? If not, why not?
c) Solve the op-amp equations to determine what this circuit does.
d) What is the maximum value for V_{in} when the potentiometer is set to its maximum resistance?

2) Another important op-amp circuit is the following:



a) Without solving any equations yet, what does your intuition tell you this circuit will do, given that a capacitor has been inserted in the feedback loop?
b) Solve the op-amp equations for a **step** input voltage.
c) How do these relate to the equations of P2.2?

3) **Hard**. This problem is hard, so it will be for extra credit. The problem is based on the following circuit:



a) What is the positive input voltage? Why do you think it was set to this voltage?
b) If D2 were a short and D1 was open, what would this op-amp do?
c) When is it true that D1 will behave as an open circuit and D2 as a closed circuit (which voltages must be higher than which ones)?
d) What does the circuit do?