16.512, Rocket Propulsion
Prof. Manuel Martinez-Sanchez
Lecture 11: Radiation Heat Transfer and Cooling

Radiative Losses

At throat of a RP1-LOX rocket, evaluate radiation heat flux

$P_c = 70\,atm$
$D_c = 0.21\,m$
$T_c = 3500\,K$
$O/F = 2.2$
$\gamma = 1.25$
$M = 25\ g/mol$
$x_{c_o} \simeq 0.38$
$x_{H_2O} = 0.31$
$x_{H_2} \simeq 0.14$
$x_{co_2} \simeq 0.11$

$$P_{throat} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$P_c = 38.85\,atm$
$P_{co} = 14.8\,atm$
$P_{H_2O} = 12.0\,atm$
$P_{H_2} = 5.4\,atm$
$P_{co_2} = 4.3\,atm$

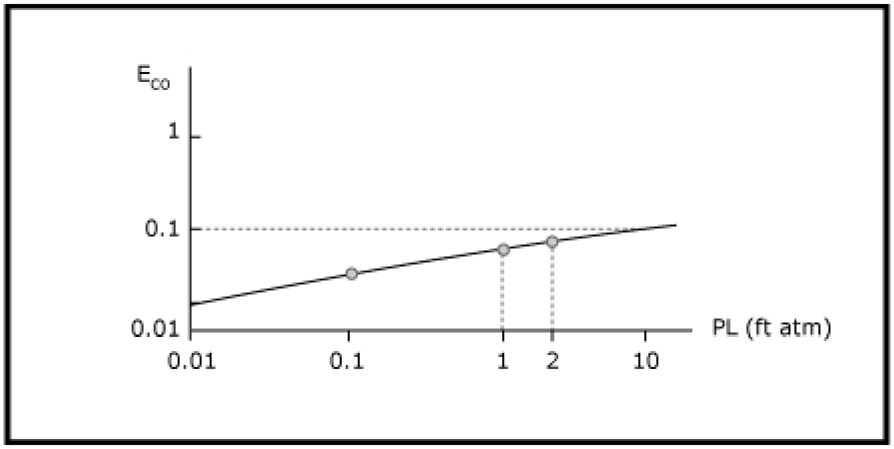
$$T_{throat} = \frac{2}{\gamma+1}T_c = 3111\,K = 5600\,R$$

Assume slab if thickness L=0.9 $D_t = 0.191\,m = 0.63\,ft$

$(PL)_{co} = 9.2\,ft\,atm$
$(PL)_{H_2O} = 7.5\,ft\,atm$
$(PL)_{H_2} = 3.4\,ft\,atm$
$(PL)_{co_2} = 2.7\,ft\,atm$

CO

Fig 4-22 for CO gas only to 2400 R
2 ft atm



So, extrapolated to 9.2 ft atm, $\varepsilon(2400\,R) \simeq 0.1$ at 2400 R. <u>But</u> ε falls rapidly with T. If we conservatively extrapolate linearly in Log ε (T). ε_{co} would appear to go to ~ 0.005 or so. Hence, even though the gas is CO-rich, radiation by CO is <u>negligible</u>.

H_2O

At $P_T = 1\,atm$, $PL = 7.5\,ft\,atm$, Fig 4.15 gives $\varepsilon_{H_2o}(5000\,R) = 0.18$, and extrapolating a bit to T=5600R, $\varepsilon_{H_2O} \simeq 0.15$.

Fig 4.15 gives ε_w for $P_w \to 0, P_T = 1\,atm$. To correct for finite P_w and higher P_T, use 4.16. Here, for $P_wL = 7.5\,ft\,atm$, there is some significant effect of $\frac{P_w + P_T}{2}$. We have $\frac{P_w + P_T}{2} = \frac{12 + 38.9}{2} = 25.5\,atm$ way beyond the graph.

$\bar{\beta} = \frac{P_w + P_T}{2}$	c_w
0.5	1 (1)
0.8	1.18 (1.14)
1	1.23 (1.21)
1.2	1.28 (1.28)
0	0.57

$$c_w = c\left(\bar{P} + B\right)^n$$

$$\begin{aligned} 0.57 &= cB^n \\ 1 &= c\left(B+0.5\right)^n \\ 1.28 &= c\left(B+1.2\right)^n \end{aligned} \quad \left. \begin{aligned} \left(\frac{B+0.5}{B}\right)^n &= \frac{1}{0.57} \\ \left(\frac{B+1.2}{B}\right)^n &= \frac{1.28}{0.57} \end{aligned} \right\} n = \frac{\ln\left(\frac{1}{0.57}\right)}{\ln\left(1+\frac{0.5}{B}\right)} = \frac{\ln\left(\frac{1.28}{0.57}\right)}{\ln\left(1+\frac{1.2}{B}\right)}$$

$$n = \frac{0.562}{\ln\left(1+\frac{0.5}{B}\right)} = \frac{0.809}{\ln\left(1+\frac{1.2}{B}\right)} \qquad \frac{\ln\left(1+\frac{1.2}{B}\right)}{\ln\left(1+\frac{0.5}{B}\right)} = 1.439 \qquad \left.\begin{aligned} B &= 0.105 \\ n &= 0.321 \end{aligned}\right\| c = 1.175$$

$$c_w = 1.175\left(\bar{P} + 0.105\right)^{0.321}$$

Then, for $\bar{P} = 25.5$, $\underbrace{c_w = 3.33}_{\text{too much}} \rightarrow \varepsilon_w = 3.33 \times 0.15 = \underline{0.499}$ Suspect!

CO_2

For PL=2.7 ft atm, T=5600 R $\varepsilon_{CO_2} \simeq 0.056$

From fig 4.14, Correction $c_c \simeq 1.1 \rightarrow \varepsilon_{CO_2} \simeq 0.062$

For interference, use Fig 4.17

$$\frac{P_w}{P_w + P_c} = \frac{12}{12 + 4.3} = 0.736$$

$$\left(P_r + P_w\right)L = 7.5 + 2.7 = 10.2 \text{ ft atm}$$

$$\Rightarrow \Delta\varepsilon \simeq 0.06$$

So $\boxed{\varepsilon_{gas} \simeq 0.499 + 0.062 - 0.06 \simeq 0.5}$

This is likely to be an over estimate, because ε_{H_2O} must saturate as P_T increases, not grow as $P_T^{0.3}$.

With this $\boxed{\varepsilon_g \sigma T_t^4 = 0.5 \times 5.67 \times 10^{-8} \times 3111^4 = 2.66 \times 10^6 \text{ W / m}^2}$

Compare to Convection:

Say $T_w = 1000\,\text{K}$ $\qquad\qquad c^* = \dfrac{\sqrt{R_s T}}{\delta}$

$$h_g = \frac{(\rho u)_{<>}^{0.8}\, c_p\, \mu_{<>}^{0.2}}{D_t^{0.2}\, P_r^{0.6}}(0.026)$$

$$c^* = \frac{\sqrt{\dfrac{8.314}{0.025} \times 3500}}{0.658} = 1640\,\text{m/s}$$

$$< T >= \frac{3111 + 1000}{2} = 2056$$

$$(\rho u)_e = \frac{P_c}{c^*} = \frac{70 \times 10^5}{1640} = 4269\,\text{Kg / m}^2\text{ / s}$$

$$(\rho u)_{<>} = 4269\,\frac{3111}{2856} = 6460\,\text{Kg / m}^2\text{ / s}$$

$$\mu_{<>} \simeq 6 \times 10^{-5}\left(\frac{2056}{3000}\right)^{0.6} = 4.8 \times 10^{-5}\,\text{Kg / m / sec}$$

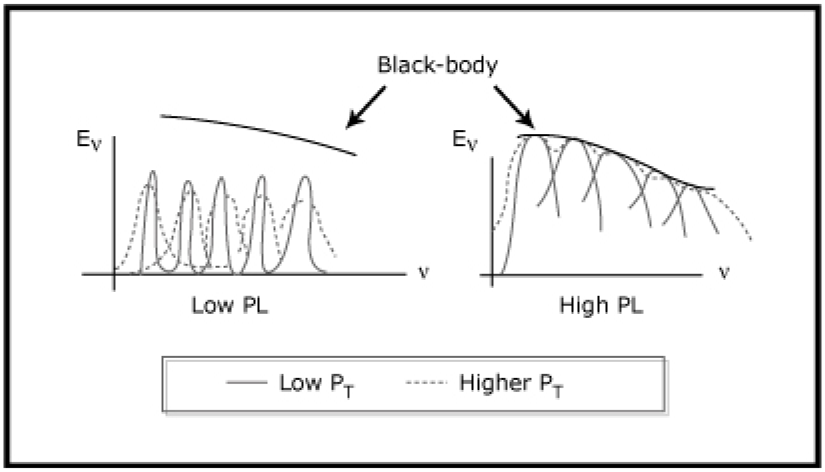
$S_p = 1663\,\text{J / Kg / K}$ $\qquad\qquad P_r = 0.8$

$$h_g = \frac{6460^{0.8} \times 1663 \times \left(4.8 \times 10^{-5}\right)^{0.2} \times 0.026}{0.21^{0.2}\ \ 0.8^{0.6}} = 345,000 \times 0.026 = 8960\,\text{W / m}^2\text{ / K}$$

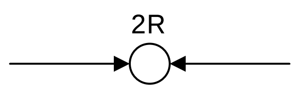
$$\boxed{(q_w)_{conv} \simeq 8960\,(3500 - 1000) = 2.24 \times 10^7 \text{ W / m}^2}$$

So $\boxed{\dfrac{q_{rad}}{q_{conv}} = 0.12}$

As P_T increases, each individual emission line is broadened by collisions, and ε increases. However, when PL is relatively large $\left(\gtrsim 2.5\,\text{ft atm}\right)$, Figure 4.14 shows the effect is small; this is because at that PL the bands are largely overlapped already and only the broadening of their edges matters anymore. So, we ignore the P_T effect.



<u>Effect of Particulates</u>



For $\frac{2\pi R}{\lambda} \gg 1$, geometrical optics

For $\frac{2\pi R}{\lambda} \ll 1$, Rayleigh regime, particle appear to be smaller by $\sim \left(\frac{2\pi R}{\lambda}\right)^4$.

For 3000 K, peak of spectrum at $\lambda \sim 1.2\,\mu\text{m}$

$$R_{\text{cross over}} = \frac{\lambda}{2\pi} \sim 0.4\,\mu\text{m}$$

Particles tend to be near (somewhat below) this value. For conservatism, assume geometrical occultation.

$$\alpha = \text{Prob. of absorption} = \text{1-Prob. of transmission} = 1 - e^{-\frac{L}{\text{mfp}}} = 1 - e^{-n_p Q_p L}$$

$$n_P = \frac{x}{1-x}\frac{\rho_{\text{gas}}}{\frac{4\pi}{3}R_p^3\rho_s} \qquad Q_p = \pi R_p^2$$

$$\boxed{\varepsilon_p \simeq 1 - e^{-\frac{x}{1-x}\frac{\rho_g}{\rho_s}\frac{3}{4}\frac{L}{R_p}}}$$

Say $L = R_t = 0.3\,m \qquad R_p = 1\,\mu m = 10^{-6}\ m$

$$\rho_g = \frac{38.9\times 10^5 \times 0.025}{8.314\times 3111} = 3.75\,Kg/m^3$$

$$\rho_s = 3000\,Kg/m^3$$

x=0.3

$$\varepsilon_p \simeq 1 - e^{-\frac{0.3}{0.7}\frac{3.75}{3000}\frac{3}{4}\frac{0.3}{10^{-6}}} = 1 - e^{-120} = 1$$

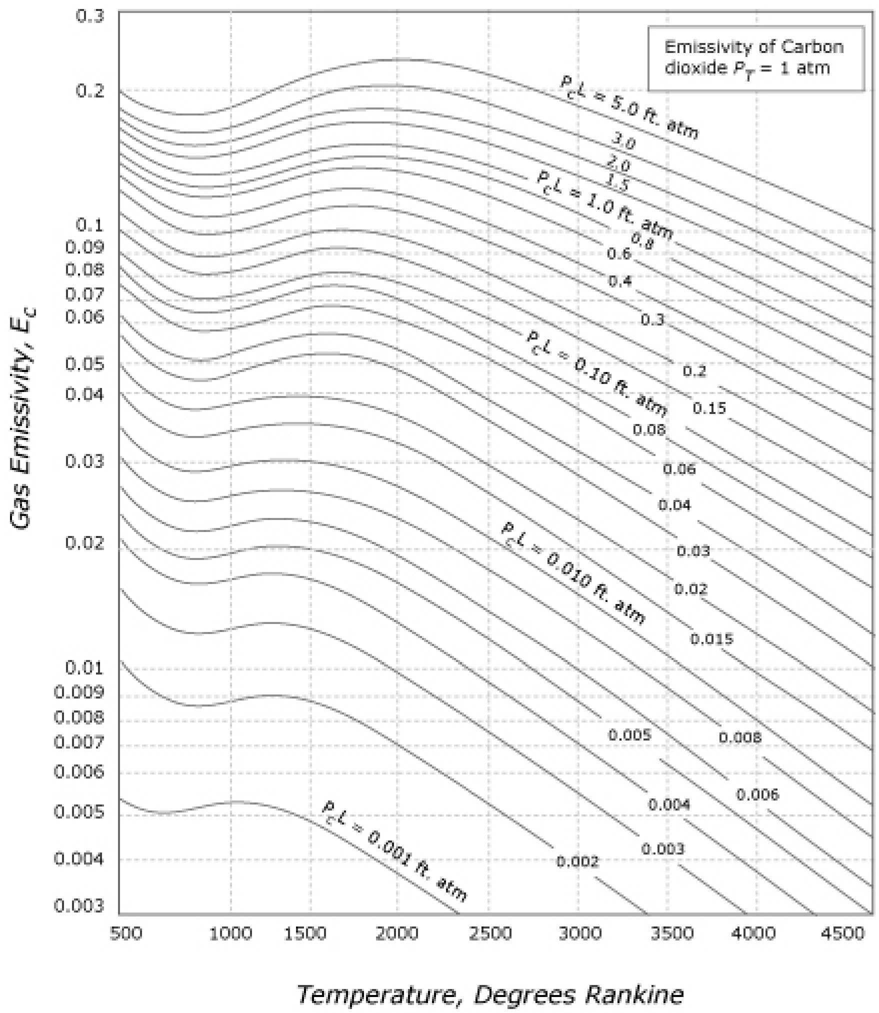
Now, suppose $R_p = 0.1\,\mu m$ instead. Exponent has a factor $\left(\frac{2\pi R_p}{\lambda}\right)^4 = \left(\frac{0.1}{0.4}\right)^4 = \frac{1}{256}$,

and has a $\frac{1}{R_p}$, which is another $\frac{1}{0.1} = 10 \rightarrow 25.6$ smaller $\rightarrow 1 - e^{-4.69} = 0.991$ still ~ 1

In this case

(a) In flame <u>looks solid</u> (black body radiator)
(b) Radiative losses <u>double</u> (ε =1 instead of 0.5), to ~20% of loss

FIGURE 4-13 EMISSIVITY OF CARBON DIOXIDE



Adapted from: Mcadams, Heat Trans.

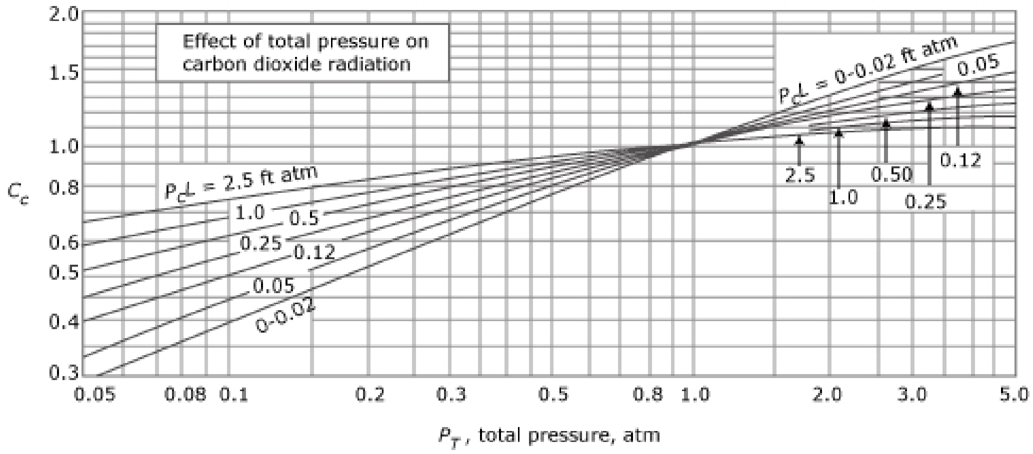
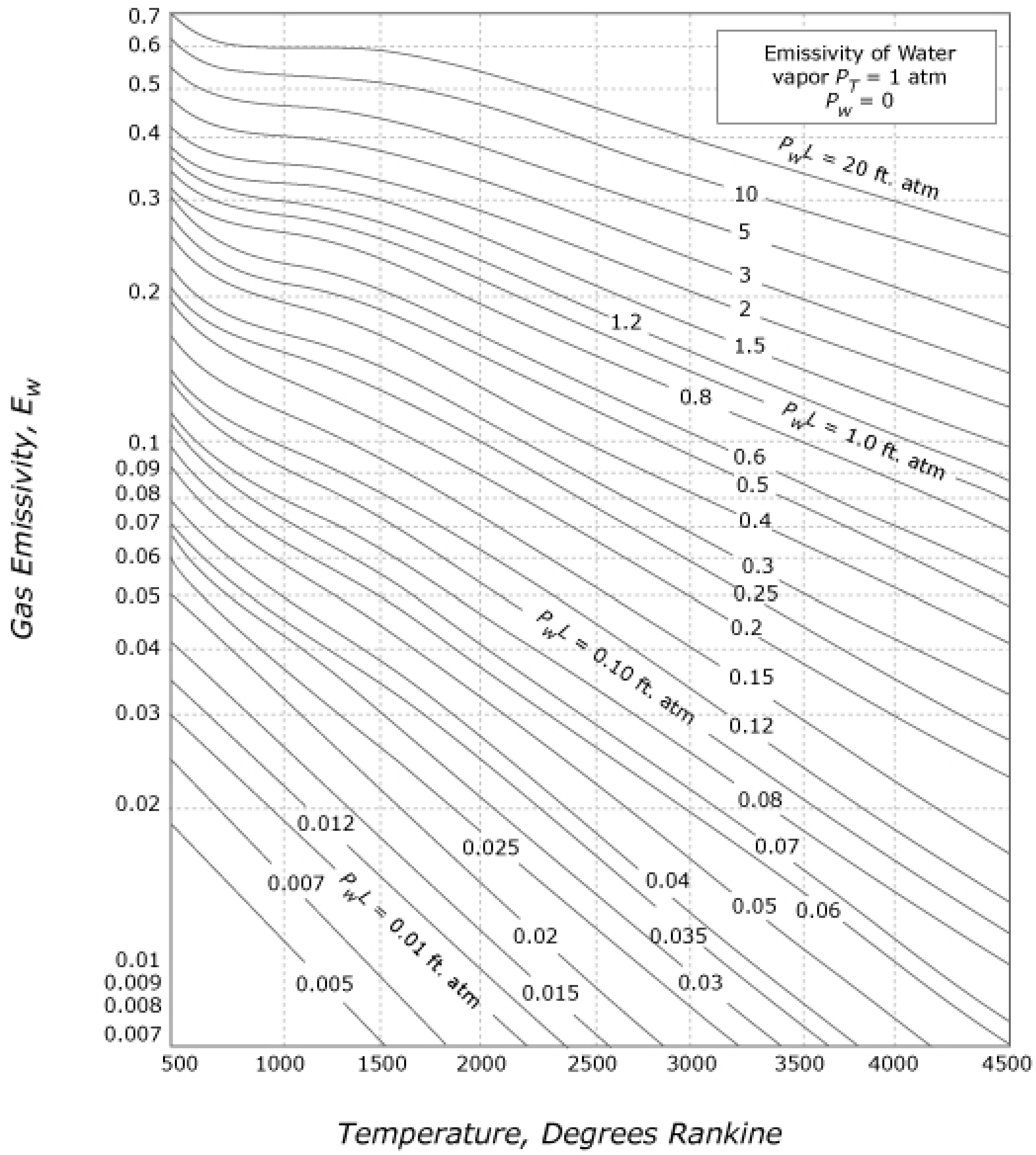


Figure 4-14 Correction factor C_c for converting emissivity of CO_2 at 1 atm total pressure to emissivity at P_T atm.

Adapted from: Mcadams, Heat Trans.

FIGURE 4-15 EMISSIVITY OF WATER VAPOR



Adapted from: Mcadams, Heat Trans.

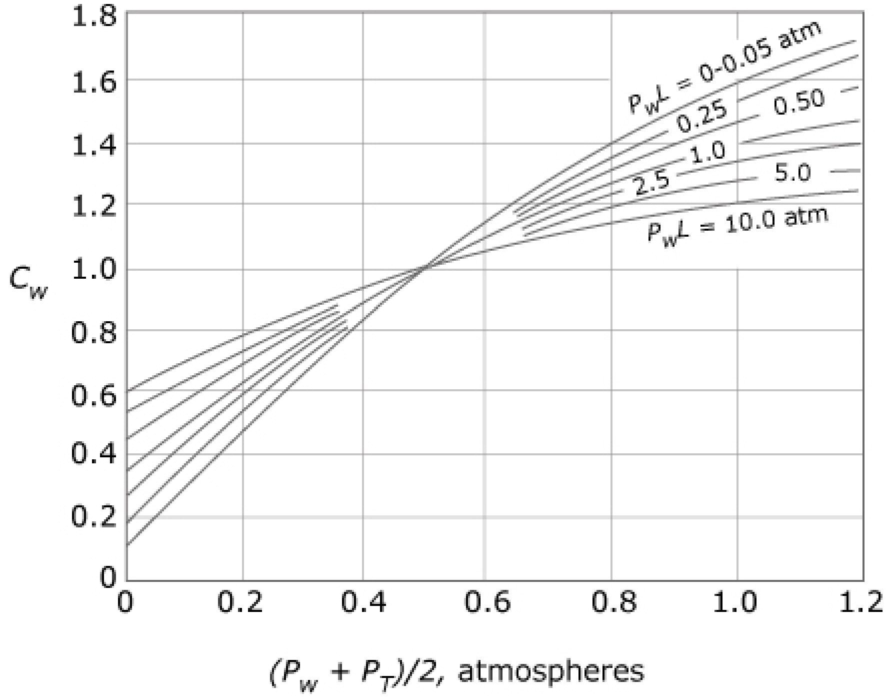


Figure 4-16 Correction factor C_W for converting emissivity of H_2O to values of P_W and P_T other than 0 and 1 atm, respectively.

Adapted from: Mcadams, Heat Trans.

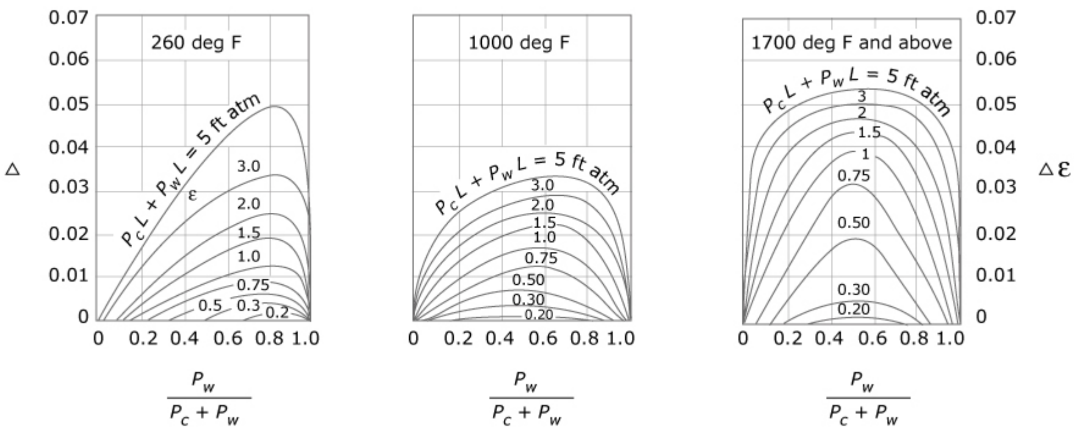


Figure 4-17 Correction to gas emissivity due to spectral overlap of water vapor and carbon dioxide.

Adapted from: Mcadams, Heat Trans.

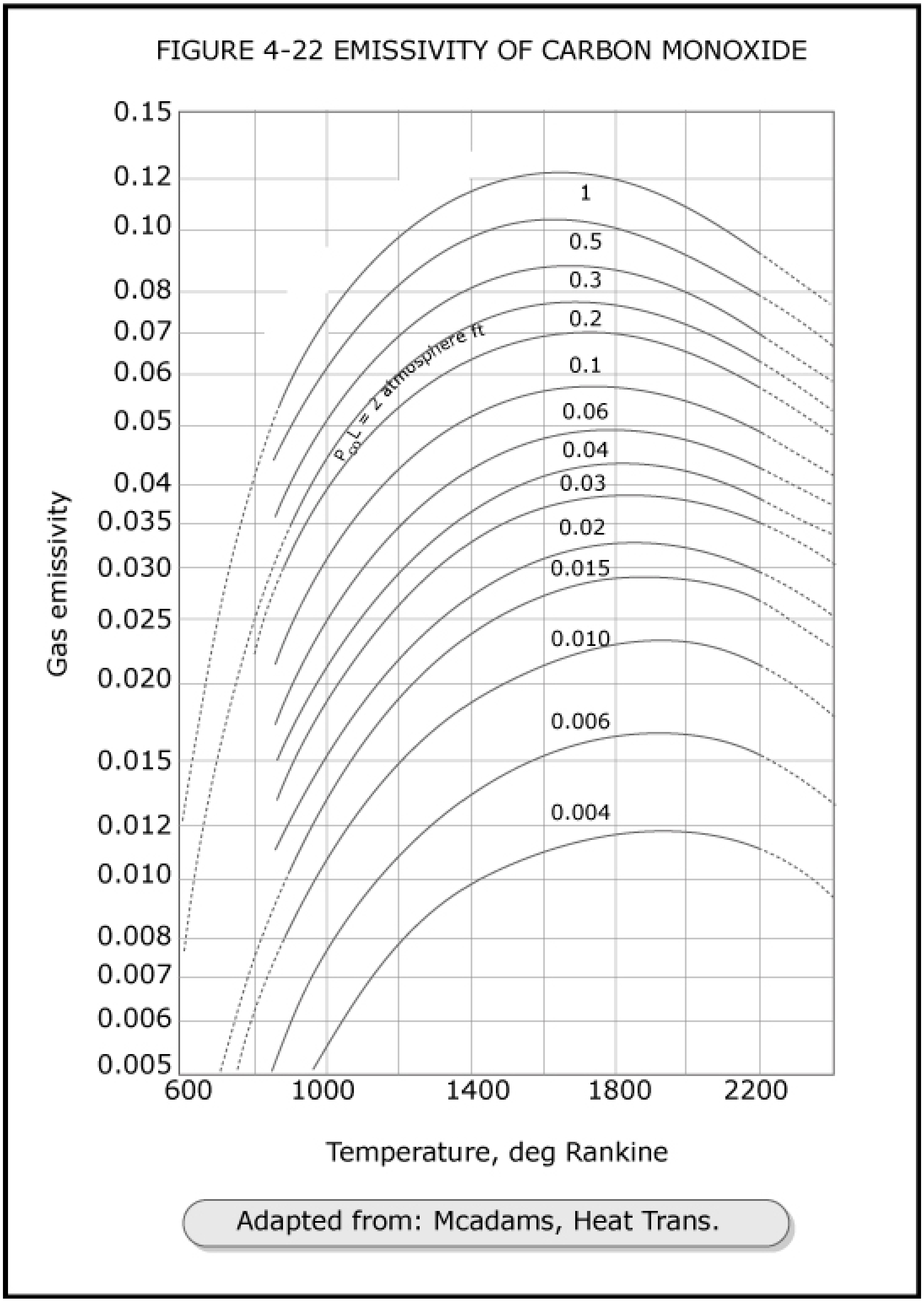
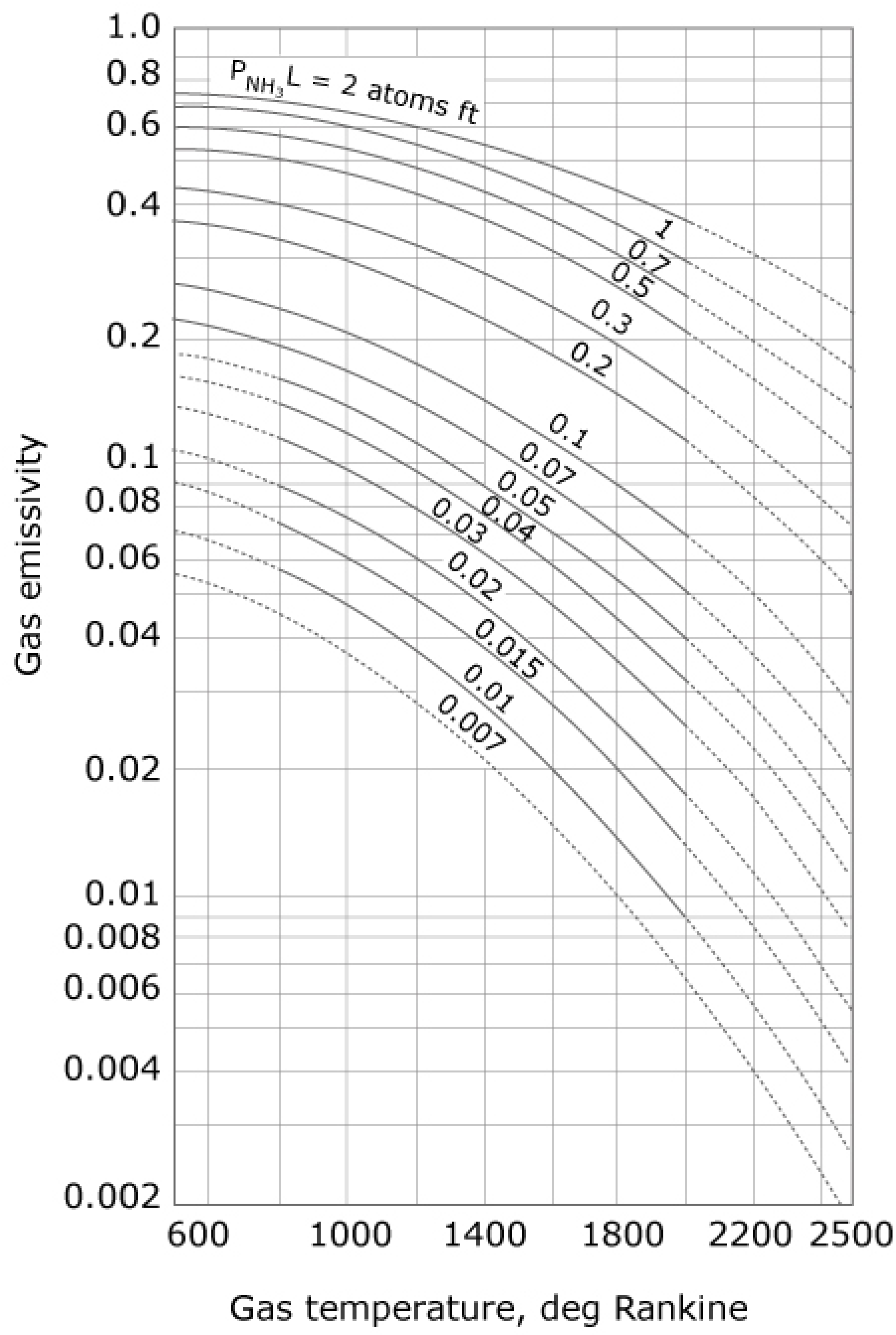


FIGURE 4-22 EMISSIVITY OF CARBON MONOXIDE
Gas emissivity
0.15
0.12
0.10
0.08
0.07
0.06
0.05
0.04
0.035
0.030
0.025
0.020
0.015
0.012
0.010
0.008
0.007
0.006
0.005
600
1000
1400
1800
2200
Temperature, deg Rankine
1
0.5
0.3
0.2
0.1
0.06
0.04
0.03
0.02
0.015
0.010
0.006
0.004
P_CO L = 2 atmosphere ft
Adapted from: Mcadams, Heat Trans.

FIGURE 4-23 EMISSIVITY OF AMMONIA



Adapted from: Mcadams, Heat Trans.