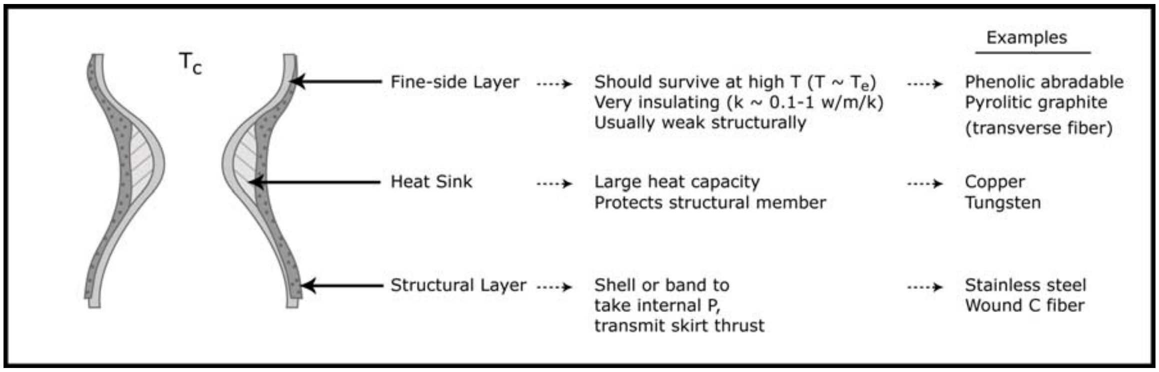
16.512, Rocket Propulsion
Prof. Manuel Martinez-Sanchez
Lecture 10: Ablative Cooling, Film Cooling

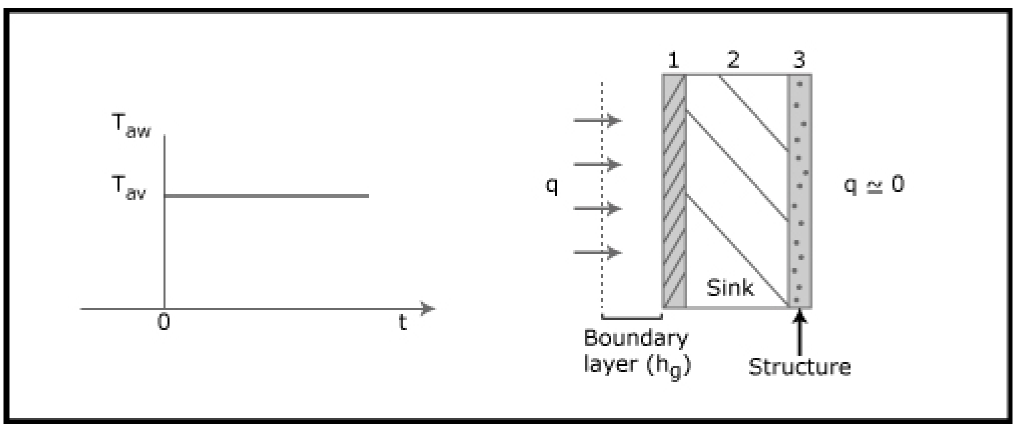
Transient Heating of a Slab

Typical problem: Uncooled throat of a solid propellant rocket



Inner layer retards heat flux to the heat sink. Heat sink's T gradually rises during firing (60-200 sec). Peak T of heat sink to remain below matl. limit. Back T of heat sink to remain below weakening point for structure.

Prototype 1-D problem:



Can be solved exactly, or can do transient 1-D numerical computation. But it is useful to look at basic issues first.

Thermal conductance of $\text{B.L.} = h_g$

Thermal conductance of front layer $= \frac{k_1}{\delta_1}$

Thermal conductance of layer i $= \frac{k_i}{\delta_i}$ ($\delta_i =$ thickness, $k_i =$ thermal conductivity)

Want layer 1 to have $\frac{k_1}{\delta_1} \ll h_g$ to protect the rest.

(Say, porous, Oriented graphyte, $\begin{pmatrix} k_1 \simeq 1\,W/m/K \\ \delta_1 = 3\,mm \end{pmatrix} \rightarrow \frac{k_1}{\delta_1} = 330 \frac{W}{m^2K}$ compared to $h_g \sim 50,000 \frac{W}{m^2K}$, so OK here).

Also, from governing equation

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \rightarrow \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

($\alpha = \frac{k}{\rho c}$, thermal diffusivity, m^2/s)

we see that

$$x^2 \sim \alpha t \text{, or } x \sim \sqrt{\alpha t} \text{, or } t \sim \frac{x^2}{\alpha}.$$

So the layer 1 will "adapt" to its boundary conditions in a time $t \sim \frac{\delta_1^2}{\alpha_1}$.

Say, $c \simeq 710 \frac{J}{KgK}$ and $\rho \simeq 1100 \frac{Kg}{m^3}$ ($\frac{1}{2}$ solid graphyte),

so $\alpha = \frac{1}{710 \times 1100} = 1.3 \times 10^{-6}\ m^2/s$.

The layer "adapts" in $t \sim \frac{\left(3 \times 10^{-3}\right)^2}{1.3 \times 10^{-6}} = 7.0\,sec$ (more like $\frac{\delta^2}{4\alpha} = 1.8\,sec$).

$\Rightarrow$ Treat front layer quasi-statically, i.e., responding instantly to changes in heat flux:

$$\boxed{k_1 \frac{T_{wh_1^{(t)}} - T_{wc_1^{(t)}}}{\delta_1} \simeq q(t)}$$

This also means we can lump the thermal resistances of BL and 1^{st} layer in series:

$$\boxed{\frac{1}{\left(h_g\right)_{eff}} \simeq \frac{1}{h_g} + \frac{\delta_1}{k_1}}$$

and since $\frac{k_1}{\delta_1} \ll h_g$,

$$\boxed{\left(h_g\right)_{eff} \sim \frac{k_1}{\delta_1} \ll} \; h_g$$

For layer 2 (the heat sink), k_2 is high (metal) and $\left(h_g\right)_{eff}$ is now small (thanks to 1st layer) so, very likely,

$$\frac{k_2}{\delta_2} \gg \left(h_g\right)_{eff}$$

(For instance, say Copper, $k_2 \simeq 360 \frac{W}{mK}$, with $\delta_2 = 2\,cm$. We now have

$\left(h_g\right)_{eff} \simeq \frac{k_2}{\delta_2} = 350 \frac{W}{m^2K}$, but $\frac{k_2}{\delta_2} = 36,000 \frac{W}{m^2K}$, so indeed, $\frac{k_2}{\delta_2} \gg \left(h_g\right)_{eff}$).

Under these conditions, the heat sink is being "trickle charged" through the high thermal resistance of layer 1. Most likely, heat has time to redistribute internally, so that T_2 is nearly <u>uniform</u> across the layer. We can then write a <u>lumped</u> equation.

$$\rho_2 c_2 \delta_2 \frac{dT_2}{dt} = q = \left(h_g\right)_{eff} \left(T_{aw} - T_2\right) \simeq \frac{k_1}{\delta_1}\left(T_{aw} - T_2\right)$$

Define $\boxed{\tau = \frac{\rho_2 c_2 \delta_1 \delta_2}{k_1}}$ $\qquad \tau \frac{dT_2}{dt} + T_2 = T_{aw} \qquad \left(T_2\left(0\right) = T_0\right)$

$$\boxed{T_2 = T_{aw} - \left(T_{aw} - T_0\right) e^{-\frac{t}{\tau}}}$$

For our example, say $\rho_2 = 8900\,Kg/m^3$ (Copper), $c_2 = 430 \frac{J}{KgK}$, $\delta_2 = 2\,cm$

$$\tau = \frac{8900 \times 430 \times 3 \times 10^{-3} \times 2 \times 10^{-2}}{1} = \underline{230\,sec}$$

This is comfortable. Suppose $T_{aw} = 3300\,K$, $T_0 = 300\,K$, and we fire for 120 sec:
(60)

$$T_2(120) = 3300 - 3000\,e^{-\frac{120\ (60)}{230}} = \underline{1520\,K}\ (989)$$

<u>May need 4 cm</u>

which is still (OK) for Copper (melts at 1360K, but no stress bearing, so can go to ~900. Also OK for steel on Carbon str member).

<u>NOTE</u>:

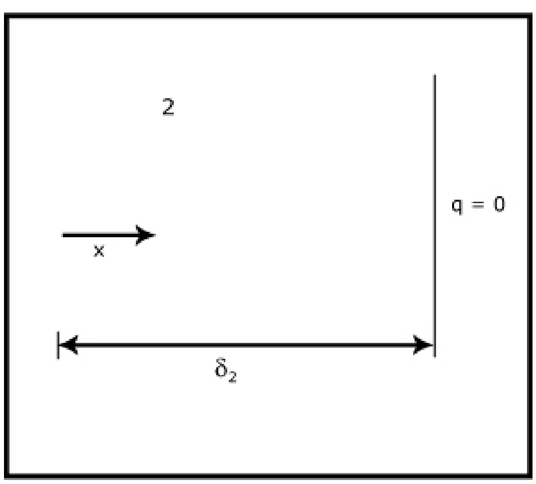
$$\frac{\delta_2^2}{4\alpha_2} = \frac{(0.02)^2}{4 \times 9.4 \times 10^{-5}} = 1.1\,\text{sec}$$, so, indeed, layer 2 “adapts” quickly to B.C.’s

$$\rightarrow \text{uniform } \frac{k_2}{\rho_2 c_2} = \frac{360}{8900 \times 430} = 9.4 \times 10^{-5}\ m^2/s.$$

<u>A More Exact Solution</u>

Consider T_{aw} “turned on” at t=0. The B.L. has a film coefficient h_g, and the first layer has δ_1, k_1, so that $(h_g)_{eff} = \dfrac{h_g}{1 + h_g \frac{\delta_1}{k_1}} \sim \dfrac{k_1}{\delta_1}$. Layer 2 has thickness δ_2, and has k_2, ρ_2, σ_2, α_2. The back is insulated.

Then one can prove that layer 2 has a temperature distribution



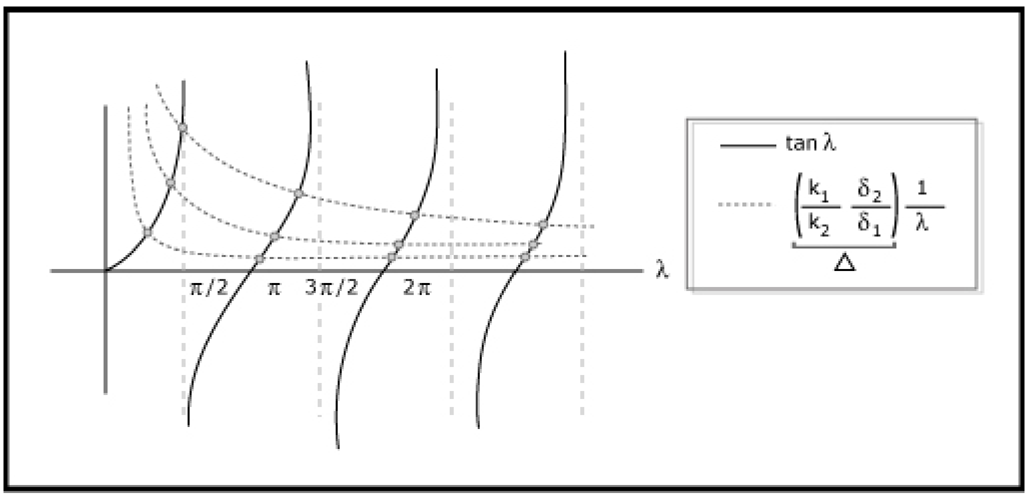
$$\frac{T_{aw} - T_2(x,t)}{T_{aw} - T_0} = \sum_{n=1}^{\infty} a_n e^{-\lambda_n^2 \frac{\alpha_2 t}{\delta_2^2}} \cos\left(\frac{\delta_2 - x}{\delta_2}\lambda_n\right)$$

where $a_n = \dfrac{2\sin\lambda_n}{\lambda_n + \sin\lambda_n \cos\lambda_n}$

and λ_n (n=1,2,...) are the roots of

$$\lambda_n \tan\lambda_n = \frac{\left(h_g\right)_{eff}\delta_2}{k_2} \simeq \frac{k_1}{k_2}\frac{\delta_2}{\delta_1}$$

Graphically,



For small $\Delta \equiv \dfrac{k_1}{k_2}\dfrac{\delta_2}{\delta_1}$, small λ_1, so $\tan\lambda_1 \simeq \lambda_1$, so

$$\lambda_1^2 \simeq \Delta \quad \lambda_1 \simeq \sqrt{\Delta} = \sqrt{\frac{k_1}{k_2}\frac{\delta_2}{\delta_1}}$$

and also $a_1 \simeq 1$ $$\lambda_1^2 \frac{\alpha_2}{\delta_2^2} \simeq \frac{k_1}{\cancel{k_2}}\frac{\cancel{\delta_2}}{\delta_1}\frac{\cancel{k_2}/\rho_2 c_2}{\delta_2^{\cancel{2}}} = \frac{k_1}{\rho_2 c_2 \delta_1 \delta_2} \equiv \tau$$

from before

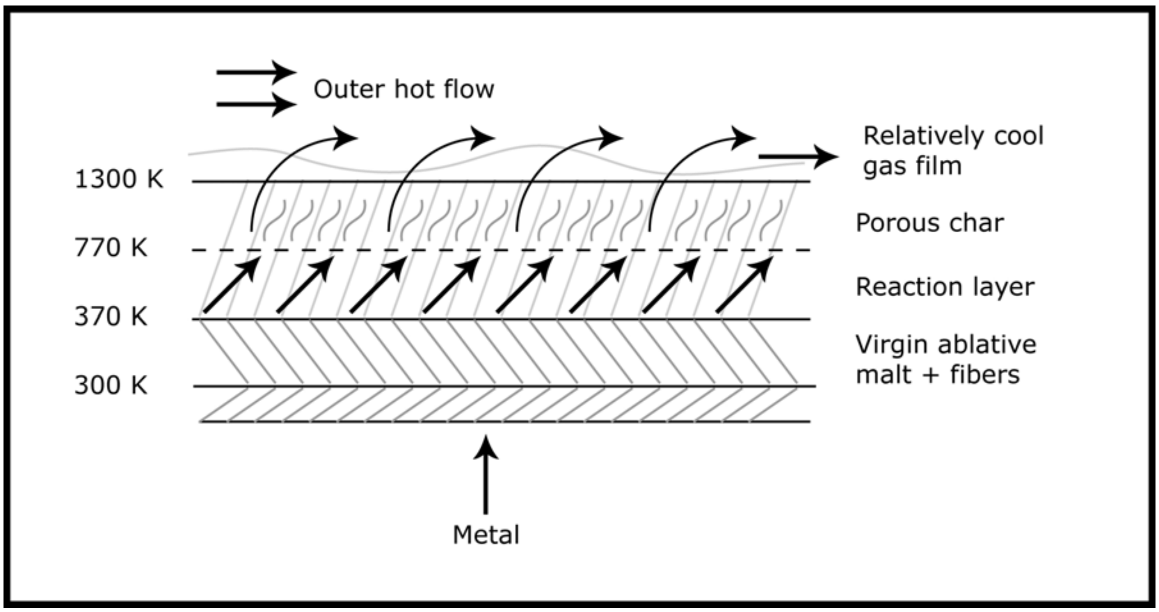
So, leading term is then

$$\frac{T_{aw} - T_2(x,t)}{T_{aw} - T_0} \simeq e^{-\frac{t}{\tau}} \underbrace{\cos\left(\frac{\delta_2 - x}{\delta_2}\lambda_1\right)}_{\simeq 1}$$

which is what we found before. The other terms are much smaller, except at very small time.

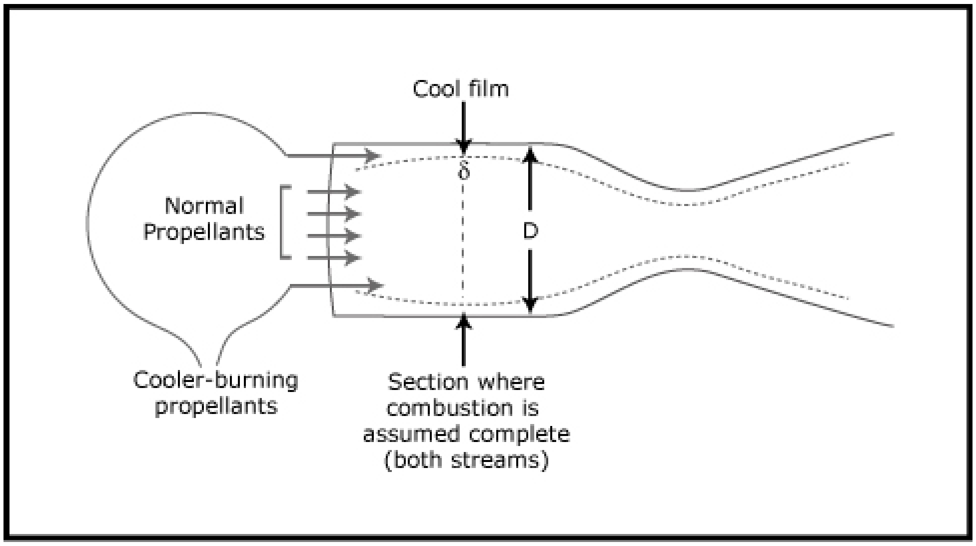
For thermal protection of solid rocket nozzles read sec. 14.2 (pp. 550-563) of Sutton-Biblarz, 7th ed., especially, pp. 556-563.

A key concept is ablative materials. They contain a C-based homogeneous matl. embedded in reinforcing fibres of strong (anisotropic) C. Best is C/C, strong expensive fibre since nozzle can get to 3600 K, can be 2D or 3D. Also good is C or Kelvin (Aramid) fibres +phenolic plastic resins (for large nozzles)

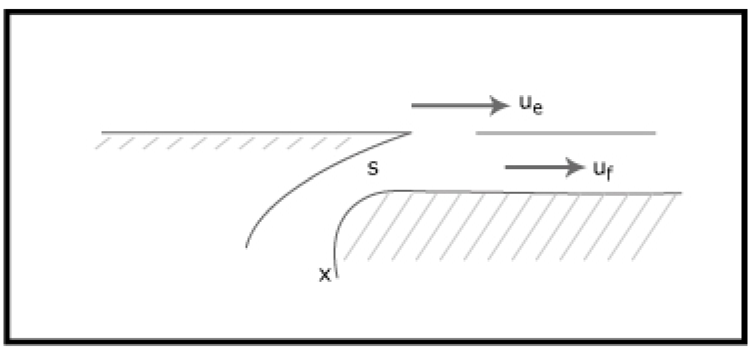


For the shuttle RSRM, the throat insert (C cloth phenolic) regresses ~ 1 inch/120 sec, and the char depth is ~ 0.5° inch/120 s.

Film Cooling of Rockets



For application of data on slot-injected films, we need to define the initial film thickness s, velocity u_F, density ρ_F, or at least mass flux $u_F \rho_F$.



Assume we know the flow rates $\dot{m}_c$ and $\dot{m}_F$, where $\dot{m}_c$ is the "core" flow and $\dot{m}_F$ the "film" flow. We also know the fully-burnt temperatures and molecular weights (T_c, T_F ; M_c, M_F).

The areas occupied at the "fully burnt" section are not known; let them be A_c, A_F. From continuity,

$$u_c A_c = \frac{\dot{m}_c}{\rho_c} = \frac{\dot{m}_c}{P} \frac{R}{M_c} T_c \qquad (1)$$

$P = P_c$ is common to both

$$u_F A_F = \frac{\dot{m}_F}{\rho_F} = \frac{\dot{m}_F}{P}\frac{R}{M_F}T_F \qquad (2)$$

and the <u>total</u> cross-section is known:

$$A_c + A_F = A \qquad (3)$$

We need some additional information to find u_F. The two momentum equations are (neglecting friction):

$$\left.\begin{aligned} \rho_c u_c \frac{du_c}{dx} + \frac{dP}{dx} = 0 \\ \rho_F u_F \frac{du_F}{dx} + \frac{dP}{dx} = 0 \end{aligned}\right\} \rho_c u_c \frac{du_c}{dx} = \rho_F u_F \frac{du_F}{dx}$$

$$\frac{u_F du_F}{u_c du_c} = \frac{\rho_c}{\rho_F} \qquad (4)$$

Both, ρ_F and ρ_c, have been evolving as drops evaporate and burn. We make now the approximation of assuming their <u>ratio</u> to remain constant (equal to the fully-burnt value). Then (4) integrates to

$$\frac{u_F^2}{u_c^2} = \frac{\rho_c}{\rho_F} \qquad \frac{u_F}{u_c} = \sqrt{\frac{\rho_c}{\rho_F}} \qquad (5)$$

Substitute into the ratio (2)/(1)

$$\frac{\rho_F u_F A_F}{\rho_c u_c A_c} = \frac{\dot{m}_F}{\dot{m}_c} \rightarrow \frac{\rho_F}{\rho_c}\sqrt{\frac{\rho_c}{\rho_F}}\,\frac{A_F}{A_c} = \frac{\dot{m}_F}{\dot{m}_c}$$

or $$\frac{A_F}{A_c} = \frac{\dot{m}_F}{\dot{m}_c}\sqrt{\frac{\rho_c}{\rho_F}} \qquad (6)$$

and also $$\frac{\rho_F u_F}{\rho_c u_c} = \sqrt{\frac{\rho_F}{\rho_c}} \qquad (7)$$

This last ratio $\left(\frac{\rho_F u_F}{\rho_c u_c}\right)$ is called the "film cooling parameter", M_F :

$$M_F = \sqrt{\frac{\rho_F}{\rho_c}} = \sqrt{\frac{M_F}{M_c}\frac{T_c}{T_F}} \qquad (8)$$

The film thickness s (at complete burn up) follows from

$$\left.\begin{aligned} A_F &= \pi\left[D^2 - (D-2s)^2\right] \\ A_c &= \pi (D-2s)^2 \end{aligned}\right\} \frac{A_F}{A_c} = \left(\frac{D}{D-2s}\right)^2 - 1 \simeq \frac{4s}{D} \quad (\text{if } s \ll D)$$

$$s \simeq \frac{D}{4}\frac{A_F}{A_c} = \frac{D}{4}\frac{\dot{m}_F}{\dot{m}_c}\sqrt{\frac{\rho_c}{\rho_F}} \qquad (9)$$

From Rosenhow & Hartnett, Chapter 17-B, we characterize film cooling by the change it induces to the driving temperature (T_{aw}) for heat flow. In the absence of a film, $T_{aw}^0 = T_c\left(1 + r\frac{\gamma-1}{2}M_c^2\right)$, and we calculate $(q_w)_{No\,Film} = h_g\left(T_{aw}^0 - T_w\right)$. The film changes T_{aw}^0 to T_{aw}^F (lower, presumably). The lowest we could T_{aw}^F to get is T_F, so we define a film cooling efficiency

$$\eta = \frac{T_{aw}^0 - T_{aw}^F}{T_{aw} - T_F} \qquad (10)$$

Limits: $\begin{cases} \eta = 0 & \text{if } T_{aw}^F = T_{aw}^0 \quad \text{(no effect)} \\ \eta = 1 & \text{if } T_{aw}^F = T_F \quad \text{(maximum effect)} \end{cases}$

If we can predict η, then

$$T_{aw}^F = T_{aw}^0 - \eta\left(T_{aw}^0 - T_F\right) \qquad (11)$$

and then

$$q_w = h_g\left(T_{aw}^F - T_w\right) \qquad (12)$$

where h_g is computed as if there were no film. To predict η, we first computes the parameter

$$\zeta = \frac{x}{M_F s}\left(Re_F \frac{\mu_F}{\mu_c}\right)^{-\frac{1}{4}} \tag{13}$$

where x is the distance downstream of the film injection (here we assume this is from the burn-out section), and

$$Re_F = \frac{\rho_F u_F s}{\mu_F} \tag{14}$$

and $\rho_F u_F = M_F\left(\rho_c u_c\right)$, from before

From ζ, there are several semi-empirical correlations for η. A recommendation from R & H is

$$\eta = \frac{1.9 P_r^{2/3}}{1 + 0.329\left(\frac{c_{p_c}}{c_{p_F}}\right)\zeta^{0.8}} \tag{15}$$

(or $\eta = 1$ if this gives >1)

which is supported by air data of Seban.

<u>Example</u>

Say $\frac{T_F}{T_c} = \frac{1}{2}$; $\frac{M_F}{M_c} = 0.8 \rightarrow \frac{\rho_F}{\rho_c} = \frac{0.8}{0.5} = 1.6 \rightarrow M_F = \sqrt{1.6} = 1.265$

$\frac{\dot{m}_F}{\dot{m}} = 0.1 \rightarrow \frac{\dot{m}_F}{\dot{m}_c} = \frac{1}{9}$
(0.01) (0.0101)

Say D=0.5m $\qquad x_t - x_{compl.comb} = 0.5\,m$

$$\left.\begin{array}{l} P = 70\ atm = 7.09\times10^6\ N/m^2 \\ T_c = 3200\,K \\ M_c = 20\,g/mol;\ \gamma_c = 1.2 \end{array}\right\}\rho_c = \frac{7.09\times10^6\times0.020}{8.314\times3200} = 5.33\,Kg/m^3;\ \rho_F = 8.53\,Kg/m^3$$

$M_c = 0.2$

$$u_c = 0.2\sqrt{1.2 \times \frac{8.314}{0.02} \times 3200} = 253\,m/s$$

$$u_F = 253\sqrt{\frac{1}{1.6}} = 200\,m/s$$

Say $\mu_F = 2 \times 10^{-5}\,Kg/m/s \to Re_F = \dfrac{8.53 \times 200 \times s}{2 \times 10^{-5}} = 8.53 \times 10^7\,s$

$$s = \frac{D}{4}\frac{\dot{m}_F}{\dot{m}_c}\sqrt{\frac{\rho_c}{\rho_F}} = \frac{0.5}{4} \times \underbrace{\frac{1}{9}}_{\text{OR } 0.0101}\sqrt{\frac{1}{1.6}} = \underset{0.000998}{0.0110\,m}$$

$Re_F = 9.37 \times 10^5$ $\left(8.51 \times 10^4\right)$

$$\frac{\mu_F}{\mu_c} = \left(\frac{T_F}{T_c}\right)^{0.6} = 0.5^{0.6} = 0.660$$

$$\zeta = \frac{0.5}{1.265 \times \underset{0.000998}{0.0110}}\left(\underset{\left(8.51\times10^4\right)}{9.37 \times 10^5} \times 0.660\right)^{-\frac{1}{4}} = \underset{(25.74)}{1.282}$$

$$\frac{c_{p_c}}{c_{p_F}} \simeq \frac{\mu_F}{\mu_c} = 0.8 \qquad (\text{say, } r_F \simeq r_c),\ P_r = 0.8$$

$$\eta = \frac{1.9 \times 0.8^{2/3}}{1 + 0.329 \times 0.8 \times \underset{(25.74)^{0.8}}{1.282^{0.8}}} = \underset{0.368}{1.24} \to \eta = \underset{0.361}{1}$$

So, this offers perfect film cooling, meaning

$$T_{aw}^F = T_F = \frac{T_c}{2} = 1600\,K$$

(3200-0.361(3200-700)=2296 K)

If the wall is made of Cu, and is at $T_w = 700\,K$, the reduction in heat flow is

$$\frac{q_w^F}{q_w^0} = \frac{1600 - 700}{3200 - 700} = 0.360$$

$$\left(\frac{2296 - 700}{3200 - 700} = 0.638\right)$$

which can be decisive.

(This example shows one could get good film cooling with much less than 10% flow in the film, maybe with only 2%).