16.50 Lecture 28

Subjects: Turbine solidity; Mass flow limits; Blade temperature

Solidity and aerodynamic loading:

Just as for the compressor, the blade spacing required to assure that the flow directions at the exits from the nozzles and the buckets are as intended, is determined by the aerodynamic loading limits of the blades. Instead of the Diffusion Factor that we used to characterize this loading in the compressor, it is usual to define a Zweifel Coefficient
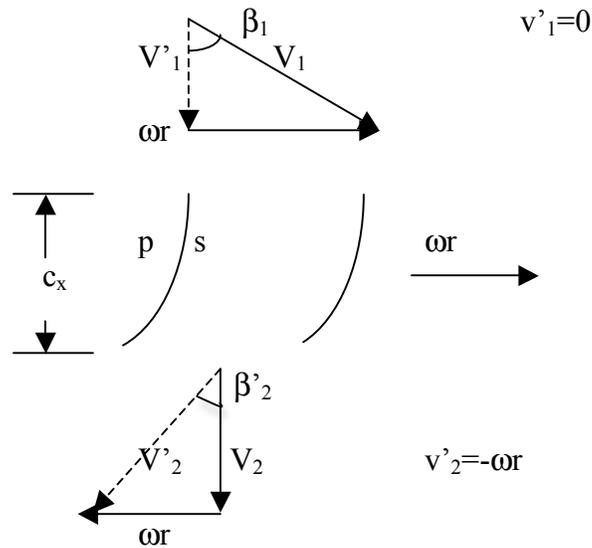
$$\psi_Z \equiv \frac{\int_{x1}^{x2}(p_p - p_s)d\frac{x}{c_x}}{p'_{t1} - p_2}$$

where x is the axial coordinate, and

          p_p = pressure on the pressure side
          p_s = pressure on the suction side

This is a measure of the pressure difference across the blade divided by the outlet dynamic pressure. In the ensuing argument we will relate it to the quantities that define the velocity triangle, just as we did for the Diffusion Factor.

From a tangential momentum balance, if s is the blade spacing and c is the axial chord,

$$\rho_1 w_1 |v'_1 - v'_2| = c_x \int_{x_1}^{x_2}(p_p - p_s)d\frac{x}{c_x}$$

Substituting in the above definition of the Zweifel Coefficient and assuming that p'$_{t2}$=p'$_{t1}$, and M^2<<1,

$$\psi_z = \frac{\rho_2 w_2 s |v_2'| \left|1 - \frac{v_1'}{v_2'}\right|}{\frac{1}{2}\rho_2 V_2'^2 c_x} \qquad\qquad \frac{w_2}{V_2'} = \cos\beta_2' \quad ; \frac{v_2'}{V_2'} = \sin\beta_2'$$

$$\psi_Z \frac{c_x}{s} = 2\sin\beta_2' \cos\beta_2' \left|\frac{v_1'}{v_2'} - 1\right| = \sin 2\beta_2' \left|\frac{v_1'}{v_2'} - 1\right|$$

which equals sin 2β'$_2$ when v'$_1$=0.

Zweifel's design rule is that loss is minimized for 0.8 < ψ$_Z$<1.0.

<u>Mass flow per unit of annulus area and blade stress</u>:

Since the pressure and hence density are higher at the inlet to the turbine than at the compressor inlet one might think that the mass flow per unit area would not be an important factor in the turbine. But it is because for a given blade speed the stress at the root of the blade increases with blade length and hence with the annulus area.

This is readily seen by computing the stress at the blade root. Consider a blade rotating as in the figure. The stress is given by
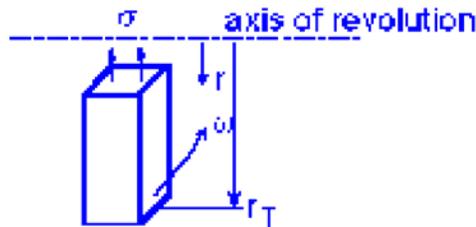

$$\sigma(r_H) = \int_{r_H}^{r_T} \rho\omega^2 r\, dr = \rho\omega^2 \frac{r_T^2 - r_H^2}{2} = \rho\frac{\omega^2}{2\pi} A_{Flow}$$

This is often quoted as the "AN^2 limit" on turbines, N being the rpm. One can see that for a given blade tip speed the stress increases as the radius of the root decreases, as is necessary to increase the flow area.

Returning to the question of the mass flow per unit area, we can compare stations a & b outside the nozzle as shown in the figure. The mass flow density is given by

$$\rho_a V_a\, H_a = \rho_b V_b\, H_b \cos\beta_b$$
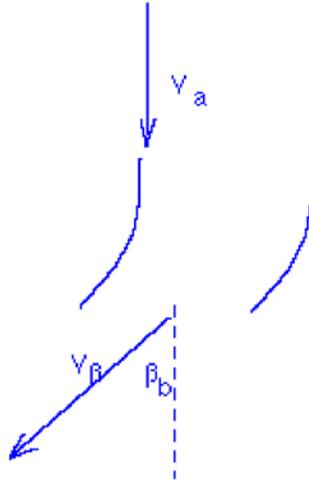
where H denotes the blade height.

The temperatures and densities at the two stations are related by

$$\frac{T_b}{T_a} = \frac{1 + \frac{\gamma\text{-}1}{2}M_a{}^2}{1 + \frac{\gamma\text{-}1}{2}M_b{}^2} \quad , \quad \frac{\rho_b}{\rho_a} = \left(\frac{T_b}{T_a}\right)^{\frac{1}{\gamma-1}}$$

and substituting these we find

$$\frac{M_a}{M_b}\left\{\frac{1+\frac{\gamma-1}{2}M_b^2}{1+\frac{\gamma-1}{2}M_a^2}\right\}^{\frac{\gamma+1}{2(\gamma-1)}} = \frac{H_b\cos\beta_b}{H_a} = \frac{(A_\perp)_b}{(A_\perp)_a} = \frac{H_b}{H_a}\cos\beta_b$$

Suppose $M_b = 1$, then we see that as β_b increases and therefore $\cos\beta_b$ decreases, M_a decreases, so the mass flow per unit area decreases. Recalling that we wanted large β_b in order to get large work from the turbine we see that there is a tradeoff between high work and low stresses.

Rotating blade temperature:

Another matter of interest is the <u>temperature</u> of the rotating blades, since the hotter they are the less stress they can tolerate. As we shall see, the stagnation temperature relative to the moving blades is not equal to that entering the nozzles.

Let T_{tb}' be the stagnation temperature relative to the rotating blade:

$$T_{tb}' = T_b + \frac{V_b'^2}{2c_p} = T_{tb} + \frac{V_b'^2 - V_b{}^2}{2c_p}$$

from which

$$\frac{T_{tb}'}{T_{tb}} = 1 + \frac{V_b'^2 - V_b{}^2}{2c_p T_b(1 + \frac{\gamma-1}{2}M_b{}^2)}$$

Now, from velocity triangles,

$$V_b^2 = w^2(1 + \tan^2 \beta_b); \quad V_b'^2 = w^2 + (w \tan \beta_b - \omega r)^2$$

so that, after some cancellations,

$$V_b'^2 - V_b^2 = -\omega r(\omega r - 2w \tan \beta_b)$$

We now recall that the degree of reaction can be written as $R = 1 - \dfrac{1}{2}\dfrac{w}{\omega r}\tan \beta_b$, so we can eliminate the term $w \tan \beta_b$. The result is

$$\frac{T_{tr}'}{T_{ta}} = 1 - \frac{\gamma-1}{2} M_T^2 \frac{3-4R}{1 + \dfrac{\gamma-1}{2} M_b^{2}}$$

So we find that T_{tb}' is generally lower than T_{tb}, reflecting the fact that the blade is "running away" from the stator flow, but also that it increases as R increases. Therefore for a given blade speed the impulse turbine will have a lower blade temperature than the 50% reaction turbine. This just introduces one more factor in the turbine design.

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