16.50 Homework 10 Solution

a) Calculate the degree of reaction and the flow angles at the stator and rotor exits (β_b, β'_c). Draw the velocity triangle to scale. Also, sketch the blade shapes of the stator and rotor.

$$\Psi = 1.6; \quad \phi = 0.8; \quad T_{t,a} = T_{t,4} = 1800\,K; \quad \gamma = 1.4; \quad R = 287\frac{J}{kg\,K}$$

$$R = 1 - \tfrac{\Psi}{2} = 0.2; \quad \beta_b = \tan^{-1}\left(\tfrac{\Psi}{\phi}\right) = 63.43°; \quad \beta'_c = \tan^{-1}\left(\tfrac{1}{\phi}\right) = 51.34°$$

b) Calculate the velocity (V_b) entering the rotor, and the axial velocity component, u. Then, calculate the wheel speed and the total temperature drop in the stage. Finally, calculate the total pressure ratio of the stage, Π_t.

Flow is choked at stator exit, so the Mach number at 'b' is one:

$$T_b = \frac{T_{t,a}}{1 + \frac{\gamma - 1}{2}\cdot 1^2} = \frac{1800\,K}{1.2} = 1500\,K \rightarrow V_b = \sqrt{\gamma R T_b} = 776\frac{m}{s}$$

$$u = V_b\cos(\beta_b) = 347\frac{m}{s} \rightarrow \quad (\omega r)\frac{u}{\phi} = 434\frac{m}{s}$$

From Euler's Equation:

$$\Delta T_t = \frac{(\omega r)^2 \Psi}{c_p} = \frac{434^2 \cdot 1.6}{1004.5} = 300\,K \rightarrow \tau_t = \frac{1500\,K}{1800\,K} = 0.833$$

$$\eta_{turbine} = \frac{actual\ work\ extracted}{ideal\ work\ extracted} = \frac{T_{t,4} - T_{t,5}}{T_{t,4} - \left(T_{t,5}\right)_{ideal}} = \frac{1 - \tau_t}{1 - \Pi_t^{\frac{\gamma-1}{\gamma}}} \rightarrow \Pi_t = \left(1 - \frac{1 - \tau_t}{\eta_{turbine}}\right)^{\frac{\gamma}{\gamma-1}} = 0.50$$

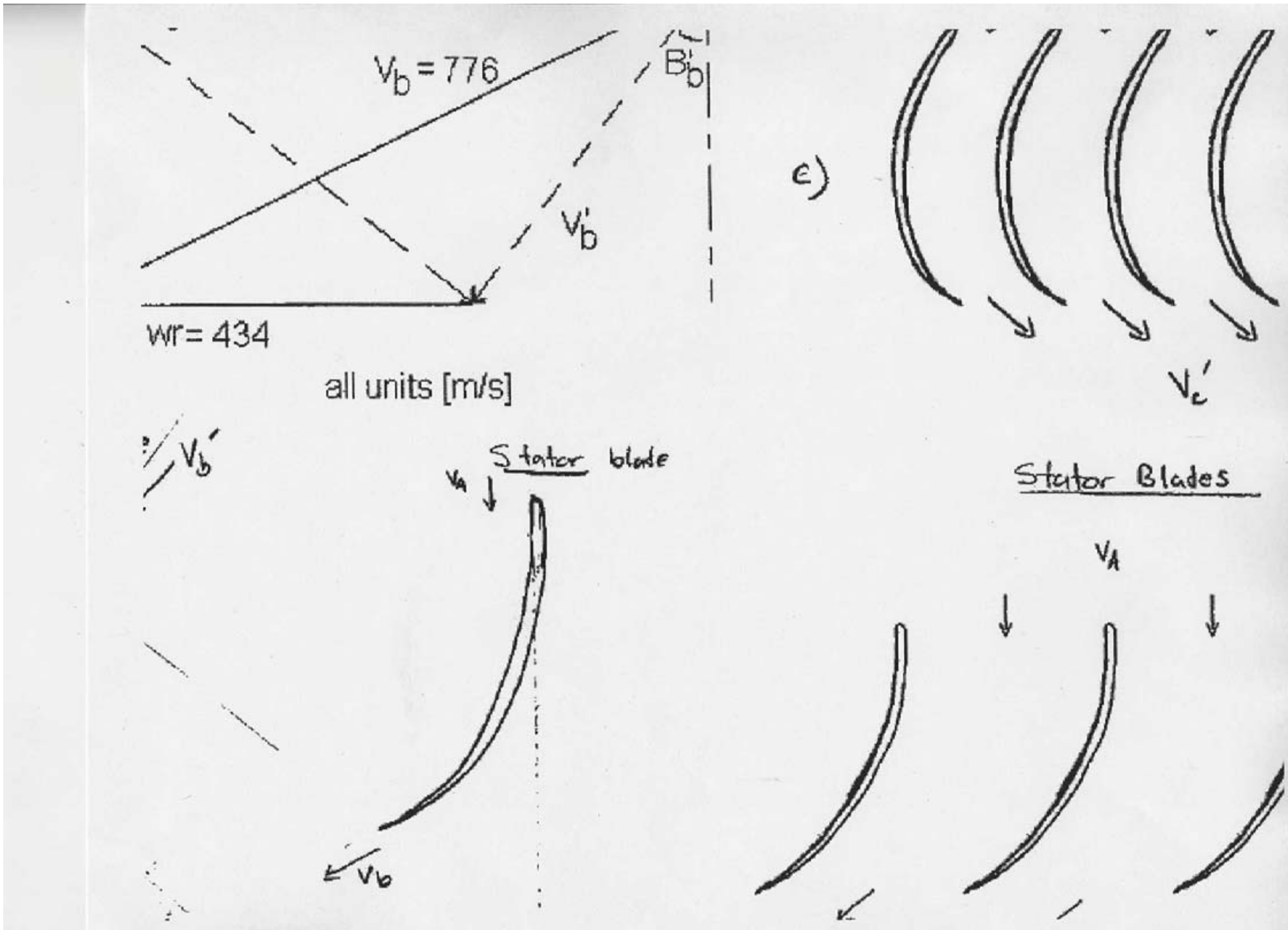
(c) Calculate the axial solidity ratio of the stator and rotor so that the Zweiffel coefficient is 0.8. Sketch a few blades each of the stator and rotor, assuming the same axial chord in both cases.

Stator: $$\left(\frac{c_x}{s}\right)_s = \frac{\sin(2\beta_b)\left|\frac{v_a}{v_b} - 1\right|}{\Psi_z} = 1$$

Rotor: $$\left(\frac{c_x}{s}\right)_r = \frac{\sin(2\beta'_c)\left|\frac{v'_b}{v'_c} - 1\right|}{\Psi_z}; \quad v'_b = v_b - \omega r = \omega r(\Psi - 1); \quad v'_c = -\omega r; \quad \frac{v'_b}{v'_c} = 1 - \Psi$$

$$\left(\frac{c_x}{s}\right)_r = \frac{\sin(2\beta'_c)|1 - \Psi - 1|}{\Psi_z} = \sin(102.68°)\frac{1.6}{0.8} = 1.951$$

Thus, for equal axial chord lengths, the rotor blades are spaced almost twice as close together as the stator blades.



Vb = 776
B'b
V'b
wr= 434
all units [m/s]
c)
Vc'
Vb'
Stator blade
VA
Vb
Stator Blades
VA