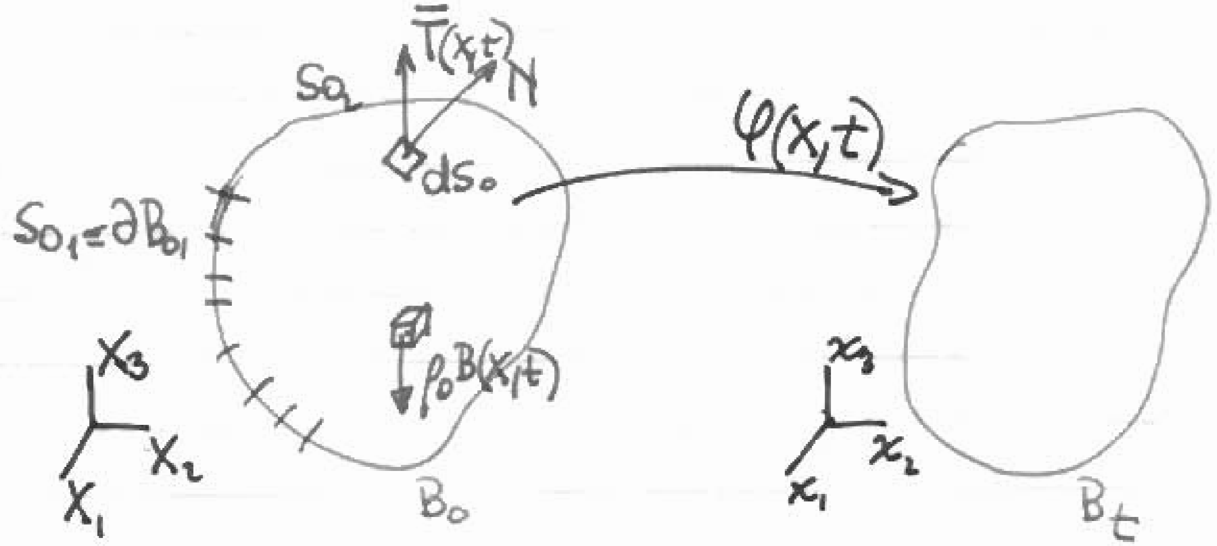
Time dependent problems

1) Nonlinear elastodynamics (hyperbolic)
2) Nonlinear heat conduction (parabolic)



Initial Boundary value problem (IBVP)

$$P_{iI,I} + \rho_o B_i = \rho_o A_i \qquad \text{in } B_o$$

$$\text{B.C.} \begin{cases} \varphi_i = \overline{\varphi}_i(X,t) & \text{on } S_{o1} \\ P_{iI} N_I = \overline{T}_i(X,t) & \text{on } S_{o2} \end{cases}$$

$$\underline{\underline{\text{I.C.}}} \begin{cases} \varphi(X,0) = \varphi_o(X) \\ V(X,0) = V_o(X) \end{cases} \qquad \begin{aligned} V(X,t) &= \varphi_{,t}(X,t) = \dot{\varphi} \\ A(X,t) &= V_{,t}(X,t) = \ddot{\varphi} \end{aligned}$$

Constitutive relations: $P = P(F, \dot{F})$

(Kelvin solid)

damping viscosity

Weak formulation: weighted residuals

$$\int_{B_0} \left[P_{iI,I} - \rho_0 (A_i - B_i) \right] \eta_i \, dV_0 = 0 \quad \forall \text{ admissible } \eta_i$$

weak form:

$$\boxed{\int_{B_0} \left[\underbrace{P_{iI} \eta_{i,I} + \rho_0 (A_i - B_i)}_{\text{two differences}} \eta_i \right] dV - \int_{S_{02}} \bar{T}_i \eta_i \, dS_0 = 0 \quad \forall \eta \text{ admissible}}$$

$P = P(F, \dot{F})$

Finite element (semi) discretization

$$(\varphi_h)_i = \sum_{a=1}^{N} x_{ia}(t) \underbrace{N_a(X)}_{\text{global shape functions}} = \sum_{e=1}^{E} \sum_{a=1}^{n} x_{ia}^e(t) N_a^e(X)$$

Introduce same interpolation for material velocity and acceleration fields:

$$\begin{cases} (V_h)_i(X,t) = \sum_{a=1}^{N} \dot{x}_{ia}(t)\, N_a(X) \\ (A_h)_i(X,t) = \sum_{a=1}^{N} \ddot{x}_{ia}(t)\, N_a(X) \end{cases}$$

Insert into weak form of linear momentum balance:

$$\begin{cases} \boxed{M\ddot{x} + f^{int}(x,\dot{x}) = f^{ext}(t)} \\ x(0) = x_0 \\ \dot{x}(0) = v_0 \end{cases}$$

where:

$$\boxed{M_{iakb} = \int_{B_0} \rho_0 N_a N_b \delta_{ik}\, dV_0 = \sum_{e=1}^{E} \int_{\Omega_0^e} \rho_0 N_a^e N_b^e \delta_{ik}\, dV_0}$$

CONSISTENT MASS MATRIX

and

$$\boxed{f_{ia}^{int} = \sum_{e=1}^{E} \int_{\Omega_0^e} P_{iI}\, N_{a,I}^e\, dV_0}$$

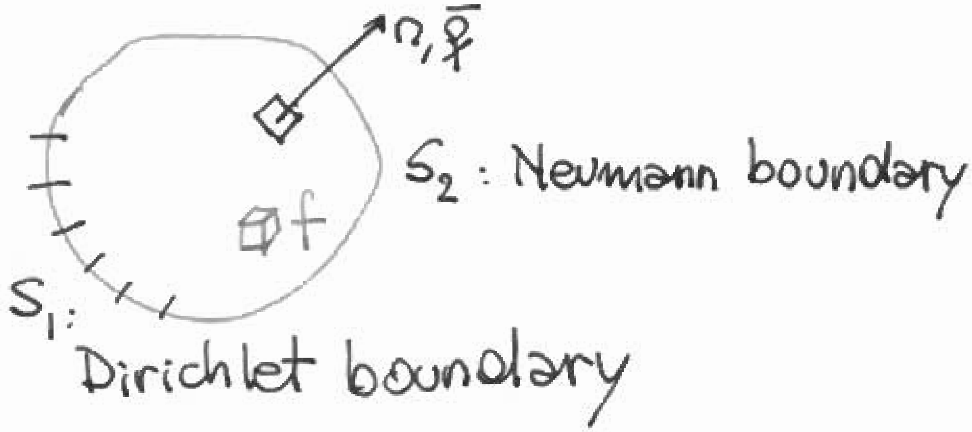
INTERNAL FORCES

$$f_{ia}^{ext} = \sum_{e=1}^{E} \left\{ \int_{\Omega_0^e} \rho_0 B_i N_a \, dV_0 + \int_{S_{0_2} \cap \partial\Omega_0^e} T_i N_a \, dS_0 \right\}$$

2) Nonlinear heat conduction

Rigid conductor, energy balance equation:

$$\begin{cases} \rho c(\theta)\, \theta_{,t} = q_{i,i} + f & \text{in } B \\ \theta = \bar{\theta} & \text{on } S_1 \\ q_i n_i = \bar{q} & \text{on } S_2 \end{cases}$$



ρ: mass density
c: heat capacity
θ: temperature
q_i: heat flux
f: heat sources