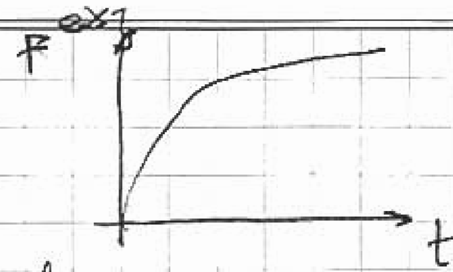
$$\boxed{F^{int}(x(t)) = F^{ext}(t)}$$



<u>Newton-Raphson solution procedure</u>

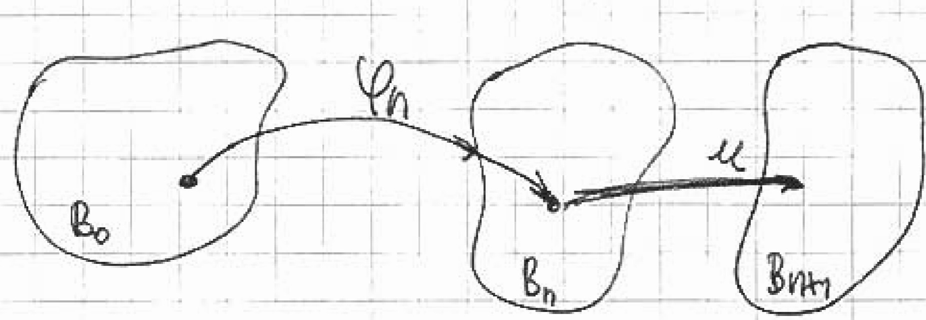
Let $t_0, t_1, \ldots, t_{n+1}$ be discrete sampling points in time. Compute $x_0, x_1, \ldots, x_{n+1}$.

Assume x_n known $\rightarrow$ want to compute x_{n+1}

$$\boxed{F^{int}(x_{n+1}) = F^{ext}(t_{n+1}) = F^{ext}_{n+1}}$$

- $x_{n+1} = x_n + u$

u: incremental displacements over B_t.



<u>continuation method</u>

$$F^{int}(x_n + u) = F^{ext}_{n+1}$$

Linearize F^{int} about x_n.

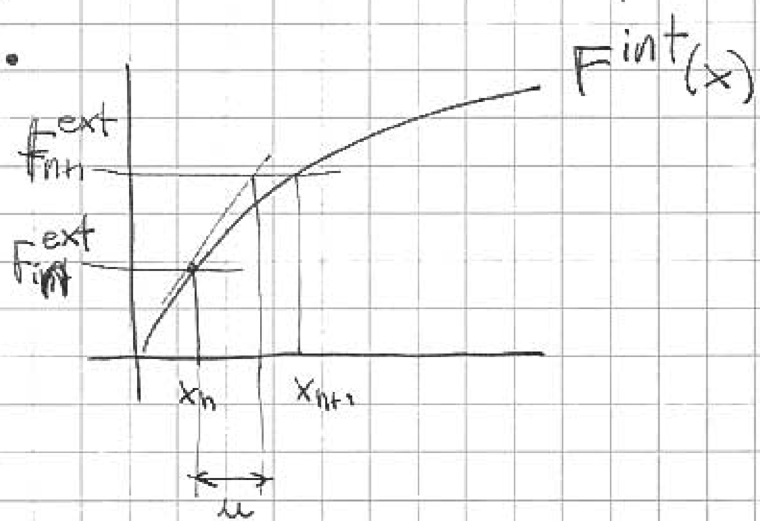
$$F^{int}(x_n + u) \simeq F^{int}(x_n) + \langle DF^{int}(x_n), u \rangle_1 + O(|u|^2)$$

$$\langle DF^{int}(x_n), u \rangle \sim F^{ext}_{n+1} - F^{int}(x_n) \equiv r_{n+1}$$

$$\equiv \text{residual force (out-of-balance)}.$$

$$\boxed{K_n(x_n)\ u = r_{n+1}} \rightarrow \text{solve for } u$$

<u>Iterate</u> the linear solution step until convergence is attained.



Iteration process:

Know x_n, F^{ext}_{n+1}

i) Initialization

$$k \to 0$$

$$x^{(0)}_{n+1} = x_n$$

ii) Compute residual

$$r^{(k)}_{n+1} = F^{ext}_{n+1} - F^{int}(x^{(k)}_{n+1})$$

iii). compute tangent stiffness

$$\longrightarrow K^{(k)}_{n+1} = DF^{int}(x^{(k)}_{n+1})$$

iv) solve $\boxed{K^{(k)}_{n+1}\, u = r^{(k)}_{n+1}} \longrightarrow u$

v) $x^{(k+1)}_{n+1} = x^{(k)}_{n+1} + u$

vi) Check for convergence

$$\| r^{(k)}_{n+1} \| \leq TOL \, \| r^{(0)}_{n+1} \| \ ?$$

yes: exit

no: $k \leftarrow k+1$, GO TO ii)

Properties

1) If $\det(K_{n+1}) \neq 0$, $\exists \rho > 0$ s.t.

if $\underbrace{\| x_{n+1}^{(0)} - x_{n+1} \| \leq \rho}_{\text{domain of attraction}} \Longrightarrow$ NR iteration converges

2) If method converges $\Rightarrow$ convergence is quadratic.

Let error $e^{(k)} = \| x_{n+1}^{(k)} - x_{n+1} \| / \| x_{n+1}^{(0)} - x_{n+1} \|$

as $k \to \infty \Rightarrow e^{(k)} \leq \alpha \left(e^{(k-1)} \right)^2 \quad \alpha \in (0,1)$

$e^{(k)} \sim 10^{-1}, 10^{-2}, 10^{-4}, 10^{-8}, 10^{-16}$

Computation of tangent stiffness

$$K_{iakb} = \frac{\partial F_{ia}^{int}}{\partial x_{kb}} = \frac{\partial^2 J_h}{\partial x_{ia}\, \partial x_{kb}} \Longrightarrow K^T = K$$

$$F_{ia}^{int} = \sum_{e=1}^{E} \int_{\Omega_0^e} P_{iJ}(F_h)\, N^e_{a,J}\, dV_0$$

$$(F_h)_{kL} = \sum_{a=1}^{N} x_{kb}\, N^e_{a,L}$$

By chain rule:

$$\boxed{\begin{aligned} \frac{\partial F_{ia}^{int}}{\partial x_{kb}} &= \sum_{e=1}^{E} \int_{\Omega_0^e} \frac{\partial P_{iJ}}{\partial F_{mN}} \frac{\partial F_{mN}}{\partial x_{kb}} N^e_{a,J}\, dV_0 \\ &= \sum_{e=1}^{E} \underbrace{\int_{\Omega_0^e} \frac{\partial P_{iJ}}{\partial F_{kL}} N^e_{a,J}\, N^e_{b,L}\, dV_0}_{K^e_{iakb}} \end{aligned}}$$

$$\boxed{K_{iakb} = \sum_{e=1}^{E} K^e_{iakb}}$$

assembly

Let $C_{iJkL}(F) \equiv$ lagrangian tangent moduli

$$K^e_{iakb} = \int_{\Omega^e_0} C_{iJkL}(F_h)\, N^e_{a,J}\, N^e_{b,L}\, dV_0$$

$$\sim \sum_{p=1}^{Q} w^e_p\, C_{iJkL}(F_h(\xi^e_p))\, N^e_{a,J}(\xi^e_p)\, N^e_{b,L}(\xi^e_p)$$

$$r_{ia} = F^{ext}_{ia} - F^{int}_{ia}(x)$$

$$F^{int}_{ia} = \sum_{e=1}^{E} \left(F^{int}_{ia}\right)^e$$

$$\left(F^{int}_{ia}\right)^e = \int_{\Omega^e_0} P_{iJ}(F_h)\, N^e_{a,J}\, dV_0$$

$$\sim \sum_{p=1}^{Q} w_p{}^e\, P_{iJ}(F_h(\xi^e_p))\, N^e_{a,J}(\xi^e_p)$$

We're considering only dead loads. Generalization

$$\bar{T}_i = \frac{\partial \phi}{\partial \varphi_i}$$ (~~the~~ for dead loads $\phi = \bar{T}_i \varphi_i$)

there are contributions to K_{iakb} (if $\bar{T}_i$ derive from potential $\rightarrow$ K still symmetric, otherwise K not symmetric.

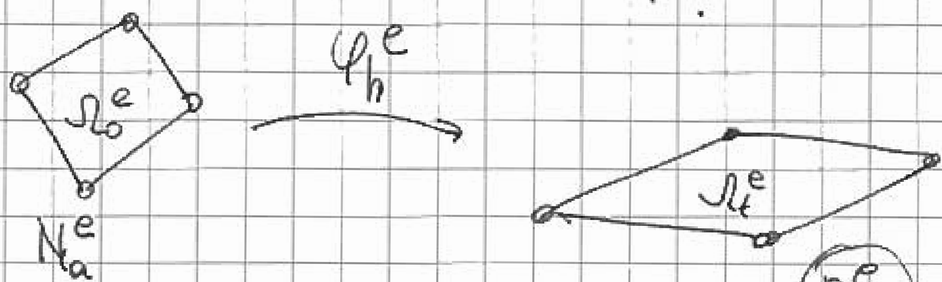
↑ LAGRANGIAN OR MATERIAL FORMULATION ↑

Spatial formulation (minor symmetries of C_{iJkL} sought).

$$F^{int}_{ia} = \sum_{e=1}^{E} \int_{\Omega_0^e} P_{iJ}(F_h)\, N^e_{a,J}\, dV_0$$

$$\sigma_{ij} = J^{-1} P_{iJ} F_{jJ} \Longrightarrow P_{iJ} = J\, \sigma_{ij}\, F^{-1}_{Jj}$$

$$F^{int}_{ia} = \sum_{e=1}^{E} \int_{\Omega_0^e} J\, \sigma_{ij}(F_h)\, \underbrace{F^{-1}_{Jj}\, N^e_{a,J}}_{?}\, dV_0$$



material shape functions

spatial shape functions

defined by composition of mappings:

$$\boxed{n_a(\varphi_h(X)) = N^e_a(X)} \qquad n_a(x) = N_a(X)$$

$$n^e_{a,j} = \frac{\partial N^e_a}{\partial X_J} \frac{\partial X_J}{\partial x_j} = N^e_{a,J} F^{-1}_{Jj}$$

also $J \, dV_0 = dV_t$

$$\boxed{F^{int}_{ia} = \sum_{e=1}^{E} \int_{\Omega^e_t} \sigma_{ij}{}_{(F_h)} \, n^e_{a,j} \, dV}$$

We note that now $\sigma_{ij} = \sigma_{ji} \Longrightarrow$

Introduce B-matrix (spatial)

$$\boxed{F^{int}_{ia} = \sum_{e=1}^{E} \int_{\Omega^e_t} B^{e^T} \sigma(F_h) \, dV}$$