16.21 Techniques of Structural Analysis and Design
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Unit #6 - Boundary value problems in linear elasticity

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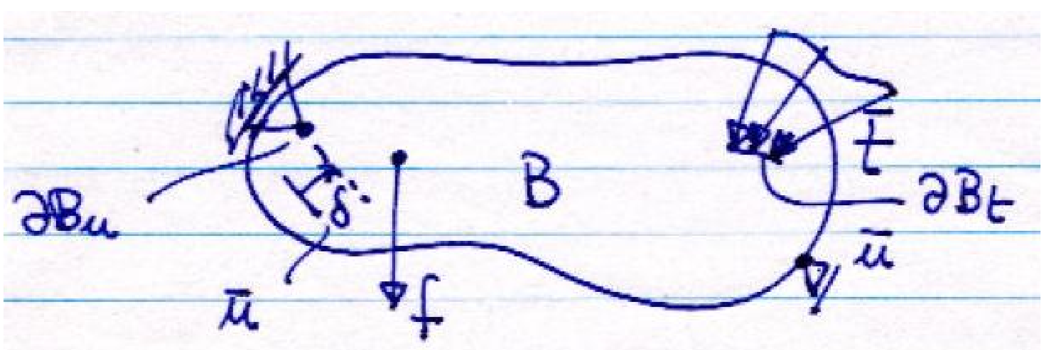

Figure 1: Schematic of generic problem in linear elasticity

- Equations of equilibrium (3 equations, 6 unknowns):

$$\boxed{\sigma_{ji,j} + f_i = 0} \tag{1}$$

- Compatibility (6 equations, 9 unknowns):

$$\boxed{\epsilon_{ij} = \frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big)} \tag{2}$$

- Constitutive Law (6 equations, 0 unknowns) :

$$\boxed{\sigma_{ij} = C_{ijkl}\epsilon_{kl}} \tag{3}$$

- Boundary conditions of two types:

  - Traction or natural boundary conditions: For tractions $\bar{\mathbf{t}}$ imposed on the portion of the surface of the body ∂B_t:

  $$\boxed{n_i\sigma_{ij} = t_j = \bar{t}_j} \tag{4}$$

  - Displacement or essential boundary conditions: For displacements $\bar{\mathbf{u}}$ imposed on the portion of the surface of the body ∂B_u, this includes the supports for which we have $\bar{\mathbf{u}} = \mathbf{0}$:

  $$\boxed{u_i = \bar{u}_i} \tag{5}$$

One can prove existence and uniqueness of the solution (the fields: $u_i(x_j),\ \epsilon_{ij}(x_k),\ \sigma_{ij}(x_k)$) in B.