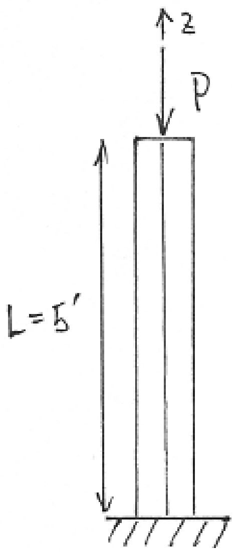
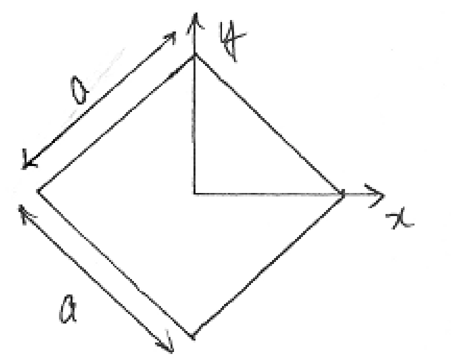
Practice Problem





$E = 10.5$ Msi

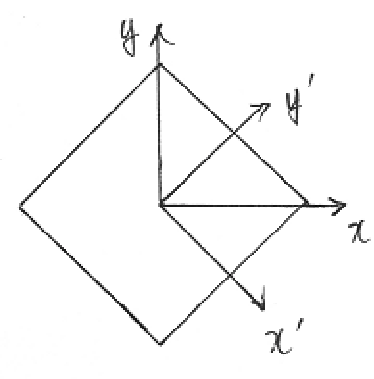
$\nu = 0.30$

$\sigma_{yield} = 42.0$ ksi

$\sigma_{ult} = 64.0$ ksi

To determine and plot the failure load as a function of the cross-sectio side length, we need to determine the buckling load (or critical load), and the yield and/or ultimate failure load. Let's deal with the buckling load first.

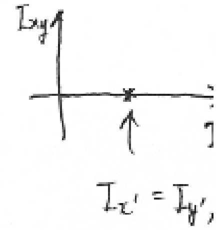
If we rotate the axis by 45° clockwise, we can calculate th moments of inertia, I_y, I_x and I_{xy} with ease because it's a square.



$$\left.\begin{aligned} I_{y'} &= \tfrac{1}{12}a^4 \\ I_{x'} &= \tfrac{1}{12}a^4 \\ I_{x'y'} &= 0 \end{aligned}\right\}$$

Mohr's circle collapses to a p

$\Rightarrow$



Thus, if we rotate the axis back to the original state, we have th same moments of inertia. This tells us that <u>there is no preferr direction for buckling</u> ($\Leftarrow$ moments of inertia in all directions are th same).

Now, let's solve for the buckling load. The governing equation is

$$EI \frac{d^4 w}{dz^4} + P \frac{d^2 w}{dz^2} = 0 \qquad \text{——— } ①$$

and the solution is

$$w(z) = A \sin \lambda z + B \cos \lambda z + Cz + D \qquad \text{——— } ②$$

$$\lambda = \sqrt{\frac{P}{EI}}$$

The boundary conditions at $z=0$ and $z=L$ are

@ $z = 0$ (clamped) :

$$w(0) = 0 \qquad \text{——— } ③$$

$$w'(0) = 0 \qquad \text{——— } ④$$

@ $z = L$ (free with axial load) :

$$\overset{\text{moment}}{EI\, w''(L)} = 0 \Rightarrow w''(L) = 0 \qquad \text{——— } ⑤$$

$$\overset{\text{shear}}{EI\, w'''(L)} = -Pw'(L) \Rightarrow w'''(L) = -\frac{P}{EI} w'(L) \qquad \text{——— } ⑥$$

* ↑ unit #16 p 14

Plugging equation ② into equations ③ through ⑥, we get the following equations.

② into ③ : $\quad w(0) = B + D = 0$

$$\Rightarrow B = -D$$

② into ④ : $\quad w'(z) = \lambda A \cos\lambda z - \lambda B \sin\lambda z + C$

$$\Rightarrow w'(0) = \lambda A + C = 0$$

$$\Rightarrow C = -\lambda A$$

② into ⑤ : $\quad w''(z) = -\lambda^2 A \sin\lambda z - \lambda^2 B \cos\lambda z$

$$\Rightarrow w''(L) = -\cancel{\lambda^2} A \sin\lambda L - \cancel{\lambda^2} B \cos\lambda L = 0$$

$$\Rightarrow A \sin\lambda L + B\cos\lambda L = 0 \qquad \text{———— ⑦}$$

② into ⑥ : $\quad w'''(z) = -\lambda^3 A\cos\lambda z + \lambda^3 B \sin\lambda z$

$C=-\lambda A$

$$\Rightarrow -\lambda^3 A\cos\lambda L + \lambda^3 B\sin\lambda L = -\frac{P}{EI}(\lambda A\cos\lambda L - \lambda B \sin\lambda L - \lambda$$

$$\Rightarrow -\cancel{\lambda^3} A \cancel{\cos\lambda L} + \cancel{\lambda^3} B \cancel{\sin\lambda L} = -\cancel{\lambda^3} A \cancel{\cos\lambda L} + \cancel{\lambda^3} B \cancel{\sin\lambda L} + \cancel{\lambda^3}$$

$$\Rightarrow A = 0 \qquad \text{———— ⑧}$$

From equations ⑦ and ⑧, we get

* Note that from $C=-\lambda A$, C
& from $B=-D$, we get $D\neq$
as well.

$$B\cos\lambda L = 0$$

$$\Rightarrow \cos\lambda L = 0 \qquad (\text{since } B \neq 0)$$

$$\therefore \lambda L = \frac{n\pi}{2}$$

$$\Rightarrow \sqrt{\frac{P}{EI}} = \frac{n\pi}{2L}$$

$$\therefore \quad P = \frac{n^2 \pi^2 EI}{4L^2}$$

$$\Rightarrow \quad P_{cr} = \frac{\pi^2 EI}{4L^2}$$

Plugging values,

$$P_{cr} = \frac{\pi^2 (10.5\,Msi)\, \frac{1}{12} a^4}{4(5 \times 12\,in)^2}$$

$$\Rightarrow \boxed{P_{cr} = 600\, a^4 \ (lb)}$$

(a in inches)

Now, let's consider the yield and ultimate loads. Since the cross-se

is constant, a^2, and force is constant,

$$P_{yield} = \sigma_{yield}\, a^2$$

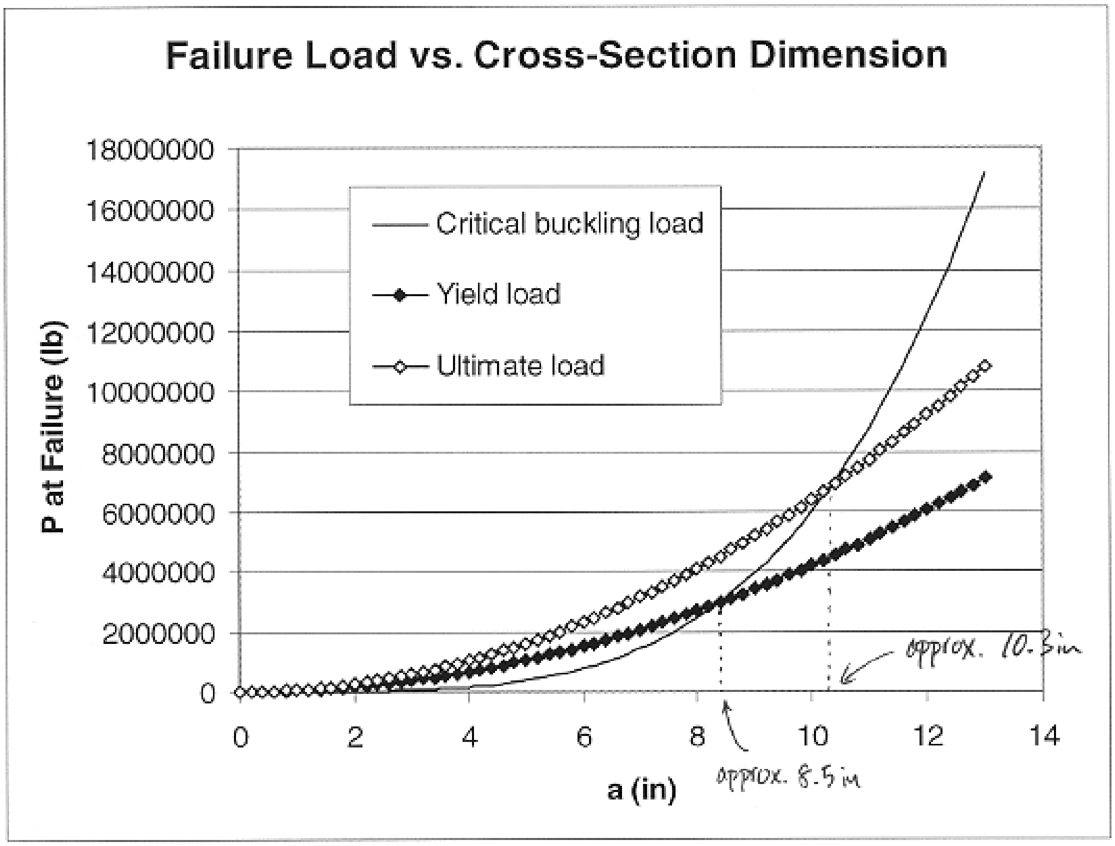
$$\Rightarrow \boxed{P_{yield} = 42000 a^2 \ (lb)}$$

$$P_{ultimate} = \sigma_{ultimate}\, a^2$$

$$\Rightarrow \boxed{P_{ultimate} = 64000\, a^2 \ (lb)}$$

The plots of the buckling load, yield load and the ultimate load ar

shown on the next page.



The plot shows that the column will likely fail due to buckling if the length of the side of the cross-section, a, is smaller than 8.5in. If a is greater than 8.5 in, yielding may occur first, and if a is greater than 10.3 in, ultimate failure will follow yielding. For values of a between 8.5in and 10.3in, a combination of yielding (which probably occurs first) and buckling would occur (in reality, due to imperfections, progressive yielding may occur, see unit #18, pp. 6~1).

This observation is also intuitive, as we would expect buckling to be mo[re] critical for long and slender columns, and squashing to be more critical fo[r] short and stubby ones.