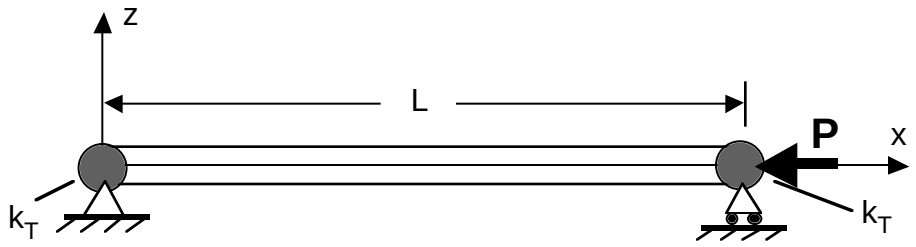
16.20 **Handed Out: Lecture 41**
Due: Lecture 46

HOME ASSIGNMENT #10

Warm-Up Exercises

A perfect column of length L has constant cross-sectional properties equal to EI. Each end of the column is held by a torsional spring of stiffness k_T. The overall configuration is depicted below with the open circle representing the torsional spring.



1. Determine the equation to be solved to find the buckling load P_{cr} (simplify as much as possible).

2. Consider the limiting case where the torsional constant goes to 0 and find the actual buckling load P_{cr}. Identify the simple configuration that has the same buckling load. Explain, from physical considerations, why this must be so.

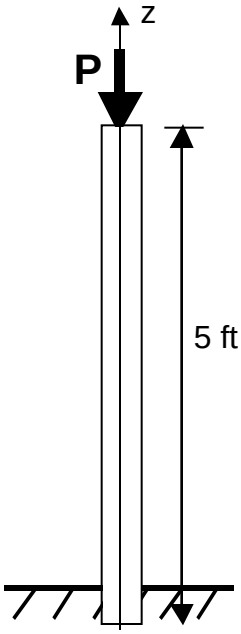
3. Consider the limiting case where the torsional constant goes to ∞ and find the actual buckling load P_{cr}. Identify the simple configuration that has the same buckling load. Explain, from physical considerations, why this must be so.

Practice Problems

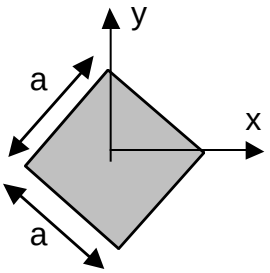
4. A perfectly loaded column is clamped at one end and free at the other where a compressive load is applied. The column is 5 feet long and has a diamond-shaped cross-section with sides of equal length a. The column is made of 2024-T3 aluminum with material properties:

$$E = 10.5 \text{ Msi}$$
$$\nu = 0.30$$
$$\sigma_{yield} = 42.0 \text{ ksi}$$
$$\sigma_{ult} = 64.0 \text{ ksi}$$

Determine and plot the failure load as a function of the cross-section side length. For buckling, indicate the direction in which buckling occurs. Label any "important points" on the plot.

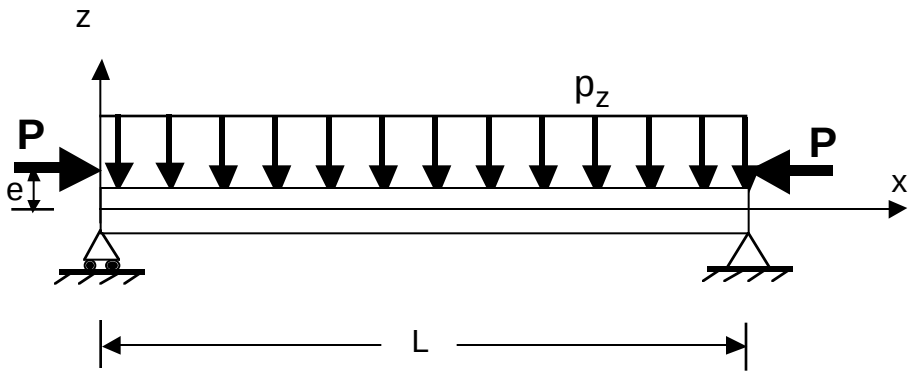


Cross-Section



Application Tasks

5. A beam-column of length L is loaded by an end load P, which is off the axis a distance e, and by a uniform load along its length p_z. The cross-sectional property EI is uniform. The structure is simply-supported.



(a) Determine the individual contributions to the primary bending moment of the transverse load p_z and the off-axis application of the end load.

(b) Using the primary and secondary moments, write the governing differential equation.

(c) Solve for the homogeneous and particular solutions and obtain the final expression for deflection w.

(d) By parametric study, determine the effects of the transverse load and the eccentricity on the mid-span deflection. Normalize the value by the length of the beam-column (w_c/L) and plot it versus load normalized by the buckling load (P/P_{cr}). (Suggestion: Choose several cases, e.g. e = 0, 0.01L, 0.05L, 0.1L; then $p_z L = 0, 0.1P_{cr}, 0.2P_{cr}, 0.5P_{cr}, 1.0P_{cr}$; then choose a couple of cross-cases. Plot the results and comment).