Lecture 26.

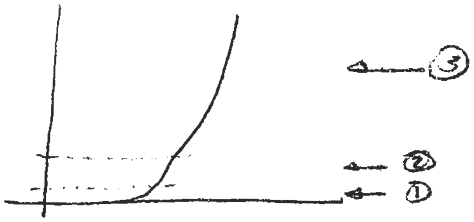
NOV 12, 03.

Turbulent Shear Layers.

7.2> A> Summary of Inner/Outer Variables
- B> Parametric effects : roughness , wall transpiration
- C> Effect Pressure gradient
- D> Equilibrium BLs.

A> Summary of Turb BL Structure

Recall turbulent BL has 3 layer structure



① Sublayer $-\overline{u'v'} << \nu \frac{\partial u}{\partial y}$ (ν imp)

② Log layer $-\overline{u'v'} \simeq > \nu \frac{\partial u}{\partial y}$

③ Wake layer $-\overline{u'v'} >> \nu \frac{\partial u}{\partial y}$

Inner Variables :

$$u^+ = y^+$$

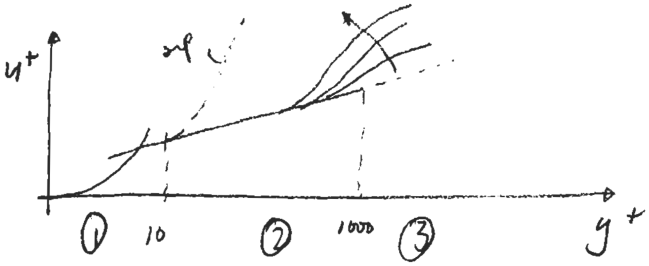
$$u^+ = \frac{1}{k} \ln y^+ + B$$

K = 0.4 — Karman const

$B \simeq 5$ (depends on roughness)

$$u^+ = \frac{u}{u^*} \qquad u^* = \sqrt{\frac{\tau_w}{\rho}}$$

$$y^+ = y \frac{u^*}{\nu}$$



Note : inner variable plot ① & ② indep of Re, dP/dx , depends on roughness other parameters

Outer

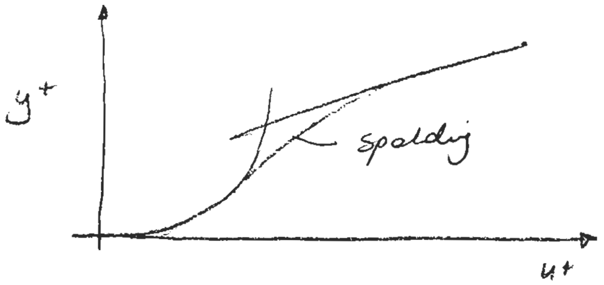
$$\frac{u_c - u}{u^*} = -\frac{1}{k} \ln (y/\delta) + A \leftarrow \text{depends on } \frac{du_e}{dx}, Re_\theta, \ldots$$

A = 2.35 wall BL , A = 0.65 pipe BL $\left(\frac{dP}{dx} = 0\right)$

Profiles in ① and ② can be written as one continuous profile

– Spalding's profile

$$y^+ = u^+ + e^{-\kappa B}\left[e^{\kappa u^+} - 1 - \overbrace{\kappa u^+ - \left(\frac{k u^+}{2}\right)^2 - \frac{(k u^+)^3}{2}}^{\text{series expansion}}\right]$$

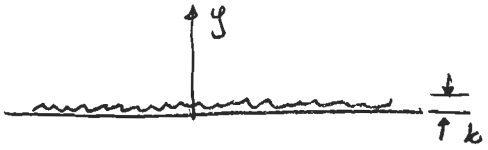


$0 \leq y^+ \leq 500$

implicit function for u^+

B> Parameter Effects

① Roughness



Non-dimensional roughness:

$$k^+ = \frac{k u^*}{\nu}$$

We can define 3 regimes based on k^+:

$k^+ < 4$ – hydraulically smooth (no effect)

$4 < k^+ < 60$ – transition roughness

$k^+ > 60$ – fully rough

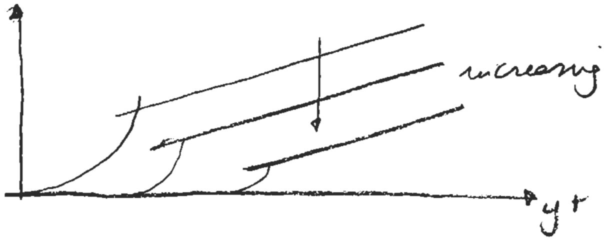
$u^+ = f(u^+, k^+)$ – sublayer

log layer

$$u^+ = \frac{1}{\kappa}\ln y^+ + \overset{\overset{5.0}{\downarrow}}{B} - \Delta u^+(k^+)$$

$$- \Delta B(k^+)$$

As roughness is increased log layer intercept B moves downward



Empirical fit of this stuff is given by

$$\Delta B = \Delta u^+ = \frac{1}{k}\ln\left(1 + 0.3k^+\right) \qquad k^+ < 60$$

for $k^+ > 60$

$$u^+ = \frac{1}{k}\ln\left(y/k\right) + 8.5$$

↑ note no effect of viscosity
fully rough flow

Note:

$$k = k^+ \frac{\nu}{u*}$$

$$\frac{k}{\delta} = k^+ \frac{\nu}{u_e\delta} \cdot \frac{u_e}{u*}$$

$$= k^+ \Big/ Re_\delta \sqrt{C_f/2}$$

Typically $C_f \approx 0.003$ $\Rightarrow$ $k/\delta \approx k^+ \frac{25}{Re_\delta}$

As Re_δ gets large, tolerable roughness height (k/δ) decreases
avg k gets smaller - requires smoother surface for same k^+

- Effect on C_f : increase with k^+, $k^+ > 60$ C_f independent of Re.
- Typical roughness k – Table 6-1 in White pg 429.

② Wall transpiration

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial \tau}{\partial y} \qquad 0 = \frac{dp}{dx}$$

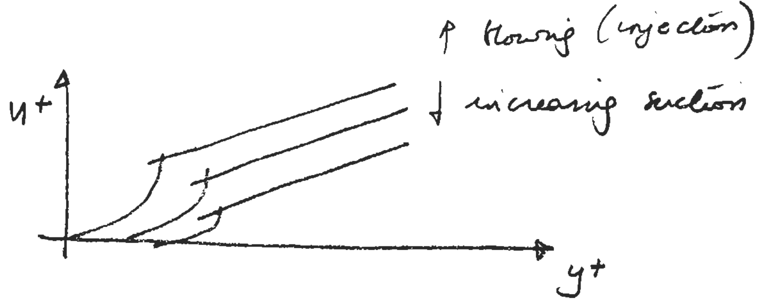
near the wall

$$\rho V_w \frac{\partial u}{\partial y} \approx \frac{\partial \tau}{\partial y} \quad \Rightarrow \quad \tau = \tau_w + \rho v_w \bar{U}$$

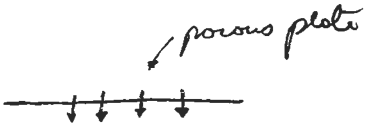
Modified log profile

$$\frac{2}{V_w^+}\left[\left(1 + v_w^+ u^+\right)^{1/2} - 1\right] \approx \frac{1}{\kappa} \ln\left(y^+\right) + B.$$

where $v^+_w = V_w / v^*$



Wake profile unaffected by weak suction



c> Effect of Pressure Gradient (Wake Profile)

Functional form: $\frac{U_c - u}{u^*} = g(y/\delta, \beta)$

$$\beta = \frac{\delta^*}{\tau_w}\frac{dp}{dx}$$

We combine log law + outer wake profile into one profile – Coles profile – wave like shape.

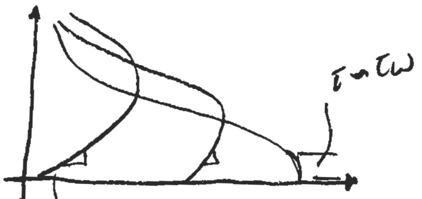
$$u^+ \approx \frac{1}{\kappa}\ln y^+ + B(\kappa, v_w^+) + \frac{2\Pi}{\kappa}W(y/\delta)$$

$$\Pi = \frac{\kappa A}{2}$$

$\Pi = \Pi(\beta)$ ← depends only

Separation: $\tau \to 0$, $l^* = \nu/u_\tau$ blows up, and assumption of log layer (triple deck) structure breaks down

At the wall



$$\frac{\partial\tau}{\partial y} = \frac{dp}{dx}$$

τ is not constant – assumption $\tau \approx \tau_w$ necessary for log layer.