4.5> Integral Methods

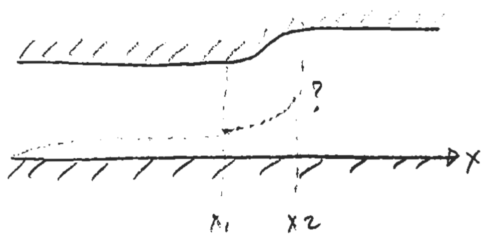
A> BL behavior example
B> Sep. behavior
C> Separation in TSL Context

Reading: Handout paper

A> BL Behavior estimate Example

Goal: Gain insight into how various terms in the 2-eqn method drive BL behavior

Problem:



$$\left(\frac{u_{e2}}{u_{e1}}\right) = 0.9$$

- sudden decrease (10%) in u_e on a flat plate BL
  - a> Will BL separate?
  - b> What is the θ increase?

Use logarithmic form of mom & K.E eqn

$$\frac{d}{dx}(\ln\theta) = \frac{1}{\theta}\, C_f/2 - (2+H)\frac{d}{dx}(\ln u_e)$$

$$\frac{d}{dx}(\ln H^*) = \frac{1}{\theta}\left[\frac{2C_D}{H^*} - C_f/2\right] + (H-1)\frac{d}{dx}(\ln(u_e))$$

To check for separation we use K.E shape param eqn.

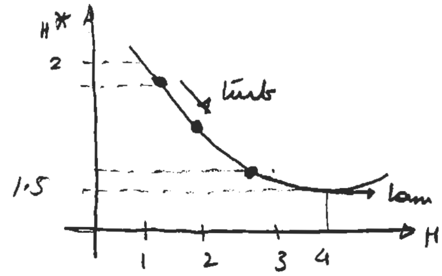
$$\int_{x_1}^{x_2}\Big\{ \qquad \Big\}\, dx$$

$$\frac{H_2^*}{H_1^*} \simeq \exp\left\{\left[\frac{2C_D}{H^*} - \frac{C_f}{2}\right]_{avg}\frac{1}{\theta}(x_2 - x_1)\right\}\cdot\left(\frac{u_{e2}}{u_{e1}}\right)^{(H-1)_{avg}}$$

Examine terms in K.E eqn → $du_e/dx < 0$ => H^* getting smaller for sufficiently fast deceleration $X_2 - X_1 \to 0$ ②

$$\Rightarrow \exp\{ \quad \} \to 1$$

$$\therefore \left(\frac{H_2^*}{H_1^*}\right) \simeq (0.9)^{(H-1)_{avg}}$$



For laminar flow

$$H_1 \simeq 2.6 \quad , \quad H_1^* \simeq 1.55$$

turbulent $\quad H_1 \simeq 1.4 \quad , \quad H_1^* \simeq 1.85$

$$\therefore \quad H_2^* \simeq (0.9)^{1.6} H_1^* = 0.84 \times H_1^* = 1.3$$ => flow will separate below $H^* \simeq 1.5$ sep. limit

$$H_2^* = (0.9)^{0.4} H_1^* = 1.78$$ => far from separation

Estimate θ_2 for turbulent flow

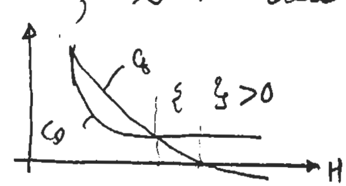
$$\ln \frac{\theta_2}{\theta_1} \simeq -(H+2) \ln 0.9$$

$$\Rightarrow \frac{\theta_2}{\theta_1} \simeq 1.43$$ sudden increase in θ / 43% increase in θ

If Δx is large, there will be additional contribution from C_f terms

For slower deceleration, the terms that multiply Δx alleviate the effects of the pressure gradient, but add to θ increase

$$\left\{ \frac{2C_D}{H^*} - C_f/2 \right\}$$

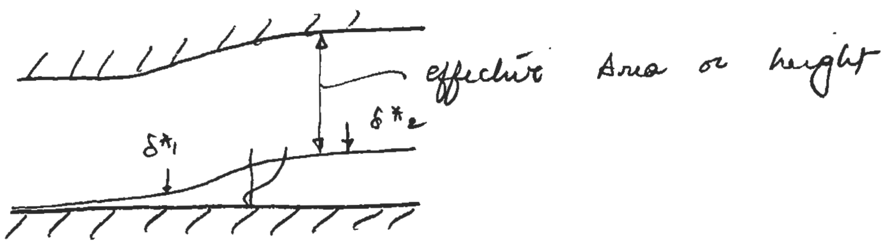


As H increases { } becomes more positive and drive H^* bigger to alleviate the tendency to separate

Going back to laminar case

$$H^* \approx 1.3$$

which is below the min "permitted" by $H^*(H)$ correlation function. In reality, the flow will separate and moderate the u_e decrease via the blockage or displacement effect (next series on IBLT) so that $H^* \rightarrow$ approach 1.5

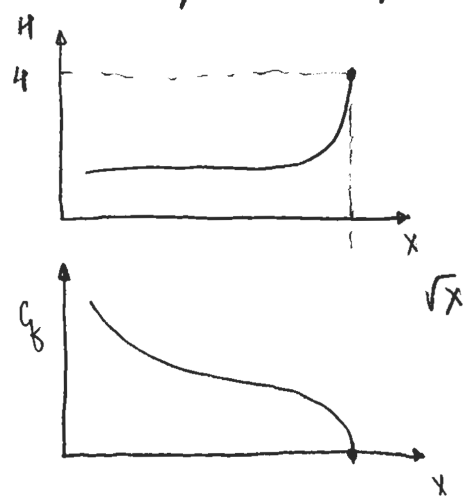


Separation Singularity

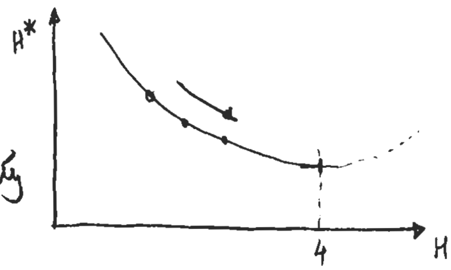
Consider a different view . . .

$$\frac{dH}{dx} = \frac{1}{dH^*/dH}\left\{ \frac{1}{\theta}\left(\frac{2C_D}{H^*} - C_f/2 \right) + (H-1)\frac{1}{u_e}\frac{du_e}{dx} \right\}$$

Towards separation point with $u_e(x)$ prescribed



$\sqrt{x}$ singularity



$$H^* \approx 1.5 + \overset{0.019}{k}(H-4)^2$$

$$\frac{dH^*}{dH} = 2k(H-4)$$

$$\frac{dH}{dx} = \frac{1}{2k(H-4)}\{ \quad \}$$

$$\Rightarrow \frac{d(H-4)}{dx} = \frac{a}{H-4}$$

$$\frac{1}{2}(H-4) = \sqrt{cx}$$

As $H \rightarrow 4$, $\frac{dH^*}{dH} \rightarrow 0$ $\therefore$ $\frac{dH}{dx} \rightarrow \infty$, $\frac{dC_f}{dx}$, $\frac{dC_D}{dx} \rightarrow \infty$

called "Goldstein singularity". Purely numerical artifact occurs (soln is infinitely sens to u_e) ill posed problem when $u_e(x)$ is imposed at separation

We can see that $\frac{dH}{dx}$ is finite only if

$$\left\{\frac{2C_D}{H^*} - \frac{C_f}{2}\right\}\frac{1}{\theta} + (H-1)\frac{1}{u_e}\frac{du_e}{dx} = 0 \quad \text{at separation}$$

or $u_e(x)$ is such that

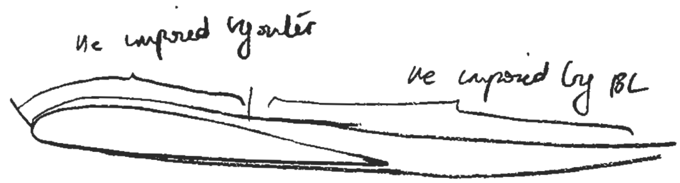
$$(H-1)\frac{\theta}{u_e}\frac{du_e}{dx} = \cancel{C_f/2}^{\to 0} - \frac{2C_D}{H^*} \quad \text{at separation}$$

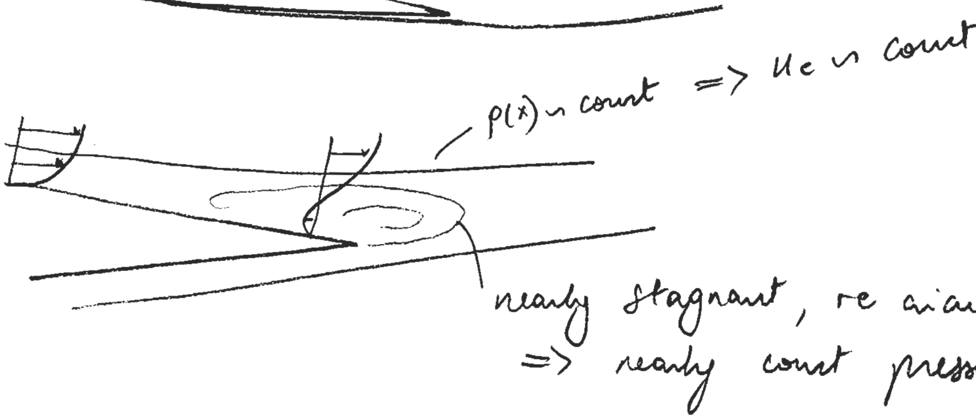
$\rightarrow$ or $\frac{du_e}{dx} \simeq -\overset{0.2}{\frac{2C_D}{H^*}}\,\frac{(u_e/\theta)}{(H-1)}$ $\leftarrow$ large

$\Rightarrow$ boundary layer determines $u_e(x)$ (in channel example via blockage (or δ^*) mechanism. ($\frac{du_e}{dx}$ is determined by C_D)

__Note__ - In other words, This requires IBLT displacement effect, so that BL can modify u_e so that $\frac{du_e}{dx}$ reaches the "admissible" value

$\frac{du_e}{dx}$ is quite small in separated flow regions





B) Separation in TSL Context

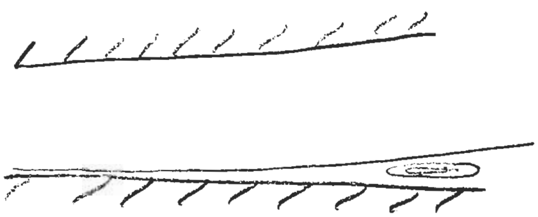
We can deal with limited separation

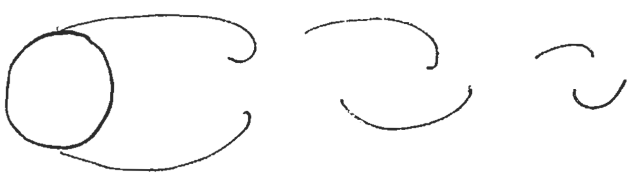
- TSL assumption (approx reasonably valid)
  - $\frac{d\delta}{dx} \ll 1$
  - $\frac{\partial p}{\partial y}$ small

1  t.e stall $\quad \frac{d\delta}{dx} \sim 0.1 \quad$ OK.

2  total or L.e stall $\quad \sim 1 \quad$ X

Large scale unsteadiness

3  diffuser sep. $\quad \sim 0.1 \quad$ OK.

4  $\sim 1.0 \quad$ X

Large scale unsteadiness.

$\Longrightarrow$ leads to IBLT lecture.

$\Rightarrow$.