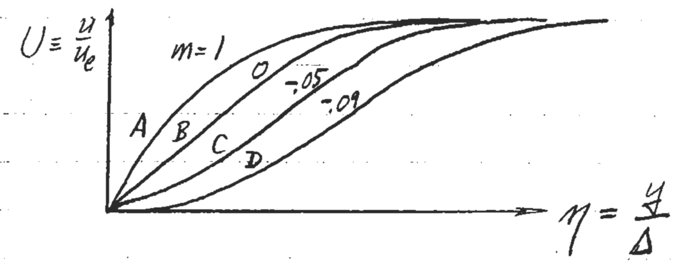
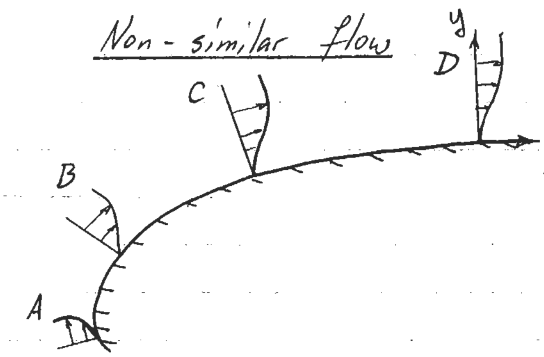
Basis for Integral BL Methods

Underlying Assumption: $u(y)$ at any x location can be fit into a profile family with suitable rescaling of u and y.

Example: Falkner – Skan profiles
$U(\eta\,;m)$ one-parameter family



Non-similar flow



To fit profile $u(y)$ at given x, we need:

- Profile parameter(s) $\quad m \equiv \frac{x}{u_e}\frac{du_e}{dx} = \frac{x}{\rho u_e^2}\left(-\frac{dp}{dx}\right)$
- Normal-length scale $\quad \Delta = \sqrt{\frac{\nu x}{u_e}} \quad$ so $\quad y = \eta \cdot \Delta$
- Velocity scale $\quad u_e \quad$ so $\quad u = U \cdot u_e$

Since x is not very relevant in non-similar flows, better locally-based choices are:

Thwaites' Method: (One-Equation Method)

Instead of $m \equiv \frac{-dp/dx}{\rho u_e^2/x}$, use $\frac{-dp/dx}{\partial\tau/\partial y} \sim \frac{-dp/dx}{\mu u_e/\theta^2} = \frac{\theta^2}{\nu}\frac{du_e}{dx} \equiv \lambda$

Instead of $\Delta = \sqrt{\frac{\nu x}{u_e}}$, use $\Delta = \int (1-U)\,U\,dy \equiv \theta$

If $u_e(x)$ is given, we need $\theta(x) \rightarrow$ Integrate $\frac{d\theta}{dx} = \frac{C_f}{2} - (H+2)\frac{\theta}{u_e}\frac{du_e}{dx}$

Still need: $H \equiv \frac{\delta^*}{\theta} = \frac{1}{\theta}\int\left(1-\frac{u}{u_e}\right)dy = \int (1-U)\,d\eta = H(\lambda)$ only

$\ell \equiv Re_\theta\, C_f/2 = \frac{\rho u_e \theta}{\mu}\,\frac{1}{\rho u_e^2}\,\mu \left.\frac{\partial u}{\partial y}\right|_0 = \left.\frac{dU}{d\eta}\right|_0 = \ell(\lambda)$ only

Two-Eq'n Method: Instead of m, use H, so $U = U(\eta\,;H)$

Now we need $\theta(x)$ and $H(x)$ $\rightarrow$ Integrate also $\frac{dH}{dx} = \frac{dH}{dH^*}\frac{H^*}{\theta}\left(\frac{2C_D}{H^*} - \frac{C_f}{2} + (H-1)\frac{\theta}{u_e}\frac{du_e}{dx}\right)$

$H^* = \int (1-U^2)\,U\,d\eta = H^*(H)$ only, $\quad Re_\theta \frac{C_f}{2} = \ell(H)$ only, $\quad Re_\theta \frac{2C_D}{H^*} = \frac{2}{H^*}\int \left(\frac{dU}{d\eta}\right)^2 d\eta = f(H)$ only