16.13 Aerodynamics of Viscous Fluids Handed out: 5 Sept 03

Problem Set 1 Due: 12 Sept 03

1) A "passive" scalar quantity s (such as the concentration of a flow tracer dye, say) which does not diffuse significantly obeys

$$\frac{Ds}{Dt} = 0$$

which simply states that s does not change at a particular point moving with the fluid.

a) Determine how the gradient of s evolves in the flow:

$$\frac{D(\nabla s)}{Dt} = ?$$

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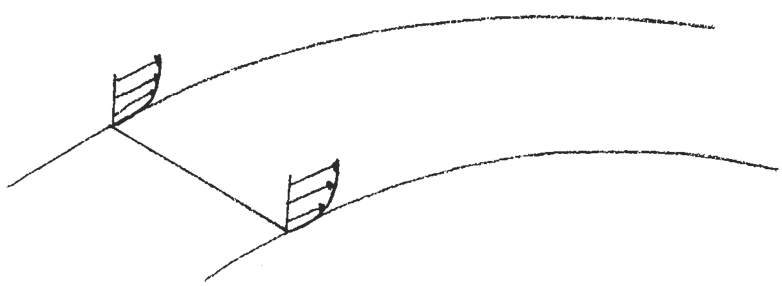
Which kinematic components of the fluid motion contribute to the changes in magnitude and/or direction of ∇s?

b) A small amount of the passive material is deposited in a flow as a small round blob, so that ∇s points radially inward to the blob center. This is then subjected to a simple 2-D shear flow.

$$\vec{u} = Ky\hat{\imath} + 0\hat{\jmath}$$

Sketch the blob boundaries, indicating ∇s, a short time later. Verify that the gradient evolved as indicated by your result from 1a).

2) The boundary layer which exists on the floor of a wind tunnel is taken through a horizontal turn in the tunnel as shown below.



3

Because the boundary layer's growth due to viscosity is relatively slow, the inviscid vorticity convection equation

$$\frac{D\vec{\omega}}{Dt} = \vec{\omega} \cdot \nabla \vec{u}$$

is approximately valid for rapid flow changes. Determine qualitatively the vorticity orientation in the boundary layer after it is turned a small amount, and the associated velocity pattern.

3) Determine which of the following equations (1) – (5) are invariant under the Galilean coordinate transformation $(\vec{r}, t) \rightarrow (\vec{r}', t')$, with $\vec{c}$ some constant frame-translation velocity.

$$\begin{aligned} \vec{r}' &= \vec{r} - \vec{c}t \\ t' &= t \\ \text{also:} \quad \vec{u}' &= \vec{u} - \vec{c} \end{aligned}$$

$$\nabla \cdot \vec{u} = 0 \qquad \text{– invariant} \tag{1}$$

$$\frac{\partial \vec{u}}{\partial t} = 0 \qquad \text{– not invariant} \tag{2}$$

4.

$$\frac{D\vec{u}}{Dt} = 0 \qquad \text{– Invariant} \tag{3}$$

$$\frac{\partial^2 \rho}{\partial t^2} - c^2 \nabla^2 \rho = 0 \qquad \text{– not invariant} \tag{4}$$