NAME : ....

Massachusetts Institute of Technology

16.07 Dynamics

Problem Set 8

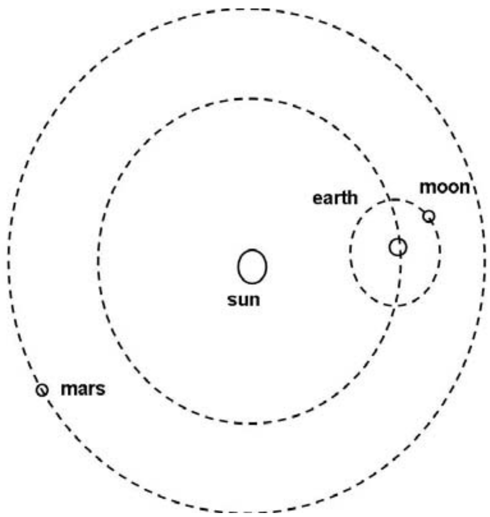
Out date: Oct 24, 2007

Due date: Oct 31, 2007

	Time Spent [minutes]
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Study Time	

Turn in **each problem** *on* **separate sheets** *so that grading can be done in parallel*

Problem 1 (10 points)



A. Determine the locations of the Lagrange points for both the earth-sun system and the earth-moon system relative to the earth. Give dimensions in km. Sketch the relative positions at some instant of time, and show as well the position of Mars.

B. Describe 2 space missions that have, will or could use the Lagrange points as key destinations. Describe the mission, and what role the Lagrange point played. If you get stuck, consider the James Webb telescope. There is also the possibility of a mission to the earth-sun L_3 point to look for the alien base. Why might they locate there?

Wikipedia will be a big help.

Problem 2 (10 points)

2-1) A spacecraft returns from the moon on the trajectory shown. We wish to place the spacecraft in a circular parking orbit at a radius $R_o = 3 \times R_e$, where R_e is the radius of the earth. Then at a later time, we wish to return the spacecraft to an elliptical orbit that includes a point B on the earth, ignoring the rotation of the earth and atmospheric drag. At the final step, we wish to bring the velocity of the spacecraft to zero and land on a non-rotating earth. The final steps–ignoring the rotation of the earth and atmospheric drag– are quite unrealistic but give a ballpark understanding. This time, do the numbers. The radius of the earth is $R_e = 6.37 \times 10^6$; the mass of the earth is $M_{earth} = 5.97 \times 10^{24}$; the universal gravitational constant is $G = 6.67 \times 10^{-11}$. Take $c = 4500 m/sec$ in the rocket equation.

2-1a) The incoming spacecraft trajectory intersects the parking orbit at an angle of 30 degrees with an incoming velocity V_i equal in magnitude to the velocity V_e required to escape the earth's gravitational field from a circular orbit radius $R_o = 3R_e$. (hint: What is the velocity required to escape from an orbit of $R_o = 3 * R_e$?) What velocity V_o will the spacecraft have in the orbit at $R_o = 3R_e$?.

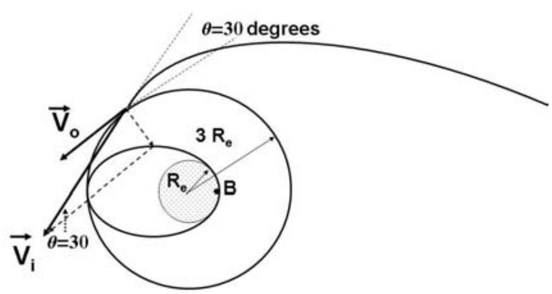
What is the ΔV required change from the initial return trajectory V_i to the parking orbit V_o?

Sketch/identify ΔV on the figure, and give an equation for the scaler magnitude of ΔV in terms of the initial velocity $\vec{V_i}$, the angle θ and the parking orbital velocity $\vec{V_o}$.

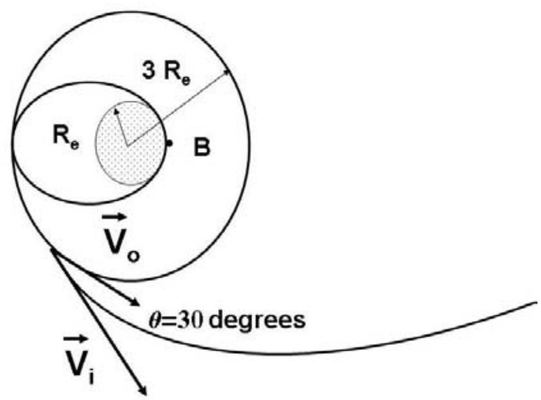
2-1b) Now, determine and apply the ΔV necessary to move from the parking orbit into an elliptical orbit which just touches the surface of the earth at point B at $r_1 = R_e$.

2-1c) Now determine the ΔV necessary to drop the velocity of the spacecraft to zero at the point B, ignoring the rotation of the earth and atmospheric drag.

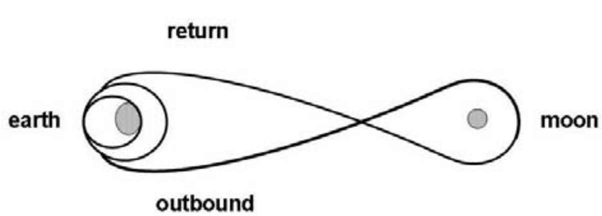
2-1d) What is the total ΔV required for this return maneuver? What percent of the vehicle mass would have to be fuel at the beginning of the return maneuver to accomplish the total maneuver? (From the rocket equation $\Delta V = -c ln(m_f/m_0)$.



2-2) Now we are going to fly the trajectory backwards, taking advantage of the image theorem. In fact, as can be seen in the sketch, if V_i and θ are chosen correctly, we will generate a trajectory that is the mirror image of the return trajectory. If we have chosen the initial conditions properly–which we have not–the sum of these two maneuvers would generate a free-return trajectory to the moon with a launch and a later return to earth. Our particular choice of θ and V_i is unlikely to be correct, but it will be close.
Essentially repeat the steps backwards to calculate $\Delta V_{outbound}$ which when added to ΔV_{return}, will give the $\Delta V mission$ for the entire mission. It will be really, really big.



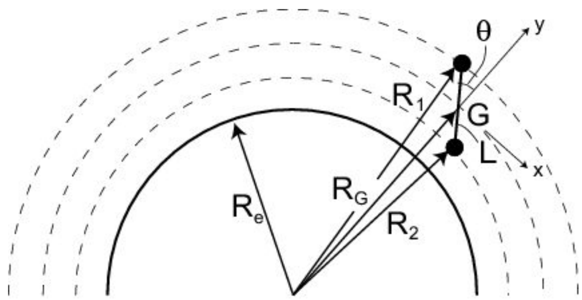
2-2a) Going backwards: determine the ΔV necessary increase the velocity of the spacecraft from zero at the point B to the orbital speed of the Homan transfer orbit, ignoring the rotation of the earth and atmospheric drag.
2-2b) Now, determine and apply the ΔV necessary to move from the elliptical orbit to the parking orbit $R_o = 3R_e$.
2-2c) Now determine the ΔV_i to launch onto the outbound trajectory.
2-2d) Determine the total ΔV for the outbound mission. What is the total ΔV required for the outbound mission? What percent of the vehicle mass would have to be fuel at the beginning of the mission to send the vehicle on its free-return trajectory to the moon? (From the rocket equation $\Delta V = -cln(m_f/m_0)$.



One final step, what is the total ΔV for the entire mission. What percent of the vehicle would have to be fuel at the beginning to accomplish the entire mission. This result is somewhat unrealistic, since we have not applied staging and we are bringing back every part of the vehicle we set out with, minus the fuel expended.

Problem 3 (10 points)

Consider a satellite in a circular orbit of radius R_G around the earth, consisting of two masses m attached by a massless structure of length $2L$. Using a rotating coordinate system fixed to the center of mass of the satellite, investigate the effects of the torque about the center of mass due to the non-uniform gravitational field in the vicinity of the satellite. Also consider the effect of varying centrifugal force on the satellite because of the different radii of the point masses. It may be surprising, but you can show that the centrifugal force produces no torque; that is there is no preferred orientation of the dumbbell due to centrifugal forces in a rotating coordinate system.



1. Recall that the gravitational force magnitude is $F = gmR_e^2/R^2$, where R is the local scalar radius to the center of the earth and R_e is the radius of the earth. However, the gravitational force is a vector pointing to the earth's center. Show that to account for both strength and direction of the gravity force at an arbitrary point, the vector representation is

$$\boldsymbol{F} = -gmR_e^2 \frac{\boldsymbol{R}}{(\boldsymbol{R}\cdot\boldsymbol{R})^{3/2}}$$

   where $\boldsymbol{R}$ is the radial vector to the arbitrary point.

2. In the rotating xyz frame, obtain vector expressions for the external gravity forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ acting on the two equal masses as well as the expression for torque about the center of mass. Express R_1 and R_2 in terms of R_G, L and θ.

3. Your expressions are complicated functions of R_G, L, θ, etc. However, typically, $L/R_G << 1$. Expand the expression for the torque for small L/R_G, and small θ. *(Retain only linear terms in L/R_G and note that $(1+x)^\alpha \sim 1+\alpha x$ for $|x|$ small and α a real number).*

   The result should be

$$\boldsymbol{M}_G = -6gmL^2 \sin\theta \frac{R_e^2}{R_G^3}\boldsymbol{k}$$

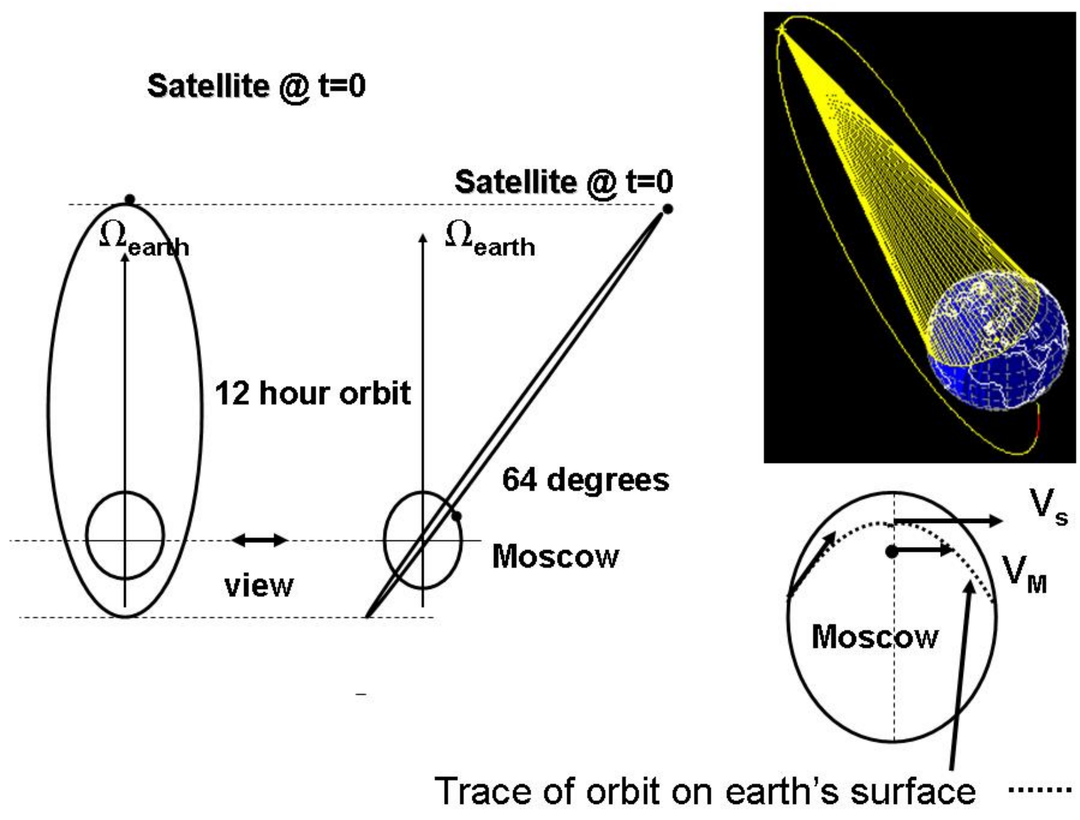
This expression may seem familiar as it is similar to the restoring torque on a pendulum. Calculate the period of the pendulum. Compare to the period of revolution of the satellite around the earth and to the Schuler period. (If you don't remember what the Schuler period is, see L13.)

4. This problem illustrates that a stabilizing, restoring force is available for free for a long, vertically-oriented satellite. Is this likely to be more effective for a low orbit or for a geosynchronous satellite?

5. This problem is related to why the moon always shows the same side to the earth. Hint: the interior mass distribution of the moon is extremely irregular. Why does the moon always show the same side to the earth?

Problem 4 (10 points)

For countries in the high northern latitudes, a geostationary communications satellite is not an attractive option because of the long distance and the low grazing angle of the communication signal. The Russians developed an alternate satellite system using an highly oblique, north-dwelling orbit called a molnyia orbit. Although a simple polar orbit could be used, for reason of communication effectiveness and stability in the presence of the oblateness of the earth, an angle of 64 degrees is commonly used for molnyia orbits. The orbit is highly eccentric. The satellite spends most of its time in the northern hemisphere at low speeds, and then comes close to the earth in the southern hemisphere at high speeds, spending a small fraction of its orbit there. The US also uses this orbit for optical satellites that spend most of their time in the northern hemisphere–I wonder why?



Determine the orbital parameters for a satellite with a 12 hour period. Ignore the rotation of the earth and the motion of the earth around the sun. In this case, the satellite will retrace its orbit twice per day. Two satellites will give reasonable coverage of the northern latitudes.

The additional criteria to be applied to fix the orbit parameters is that the satellite spends 70% of its period above the equator. If you have trouble with the geometry, just work this out for a polar (vertical) orbit. What is the semi-major axis of the orbit? How close does it come to the surface of the earth? How far away does it get? What is its eccentricity?

For a non-rotating earth, sketch the trace of the satellite's orbit, defined as the points that are directly below the satellite at various times in its orbit. (Directly below is defined as being on a line from the satellite to the center of the earth.) Take the line from the equator to the north pole through Moscow (located at latitude @ 55.5^o) as your centerline; take $t = 0$ at the time when the satellite is at its most northerly point. What is the most northerly point? The most easterly point? When does it occur? The most westerly point? When does it occur? What is the velocity of the satellite orbit's trace point at the centerline directly above Moscow? For a rotating earth, what is the velocity of Moscow?