Massachusetts Institute of Technology

16.07 Dynamics

Problem Set 7

Out date: Oct 17, 2007

Due date: Oct 24, 2007
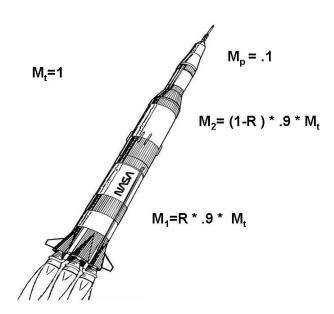
	Time Spent [minutes]
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Study Time	

Turn in **each problem** *on* **separate sheets** *so that grading can be done in parallel*

Problem 1 (10 points)

Consider the design and optimization of a 2 stage rocket. We desire to determine the optimum mass ratio between the first and second stages of a rocket. The payload is 10% of the **total** mass of the rocket M. The mass of the first and second stages together is then 90% of the total mass M. 90% of the mass of each stage is fuel. The mass of the first stage is R times the combined mass of the two stages, ($m_1 = .9 * R * M$), leaving $(1 - R)$ times the combined mass of the two stages for the mass of the second stage ($m_2 = .9 * (1 - R)M$). (R is a number less than 1, say .55 as a typical value.) Now let's launch. Take $c = 4500$ and for convenience take the total mass M equal to 1 (payload plus both stages; $M = 1$).
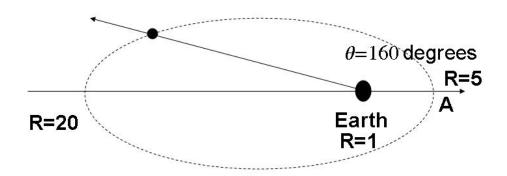
Ignore gravity and use the expression for the "ideal" Δv from a rocket firing from Lecture 14, $\Delta(v)_{ideal} = -cln(\frac{m_f}{m_0}) = -cln(\mu)$. Calculate the velocity after the first stage is fired to completion and about to be thrown away. Then calculate the additional velocity from the firing of the upper stage after the first stage has been discarded.(Using $(v_0)_{ideal}$ ignores the effect of gravity and gives the most optimistic prediction of final velocity.)

1-a) Now fire the first stage and fly it to completion. For a general value of R, and the specified masses of the payload, fuel and second stage, what is the final velocity of the rocket after the first firing? What is μ for this part of the calculation?

1-b Now fire the second stage, and fly it to completion. What is the final velocity of the rocket v_f after this firing. Express this final velocity as a function of R, the mass ratio of the first stage m_1 to the total rocket mass $m_1 + m_2$ (excluding payload). What is μ for this part of the calculation?

1-c Find the **optimum** value of R that maximizes the final velocity. You may do this by either plotting $v_{final}(R)$ vs. R, or by taking $(\frac{dv}{dR})_{final}$, setting it equal to zero, and solving for R.

1-d Compare this "**optimum**" velocity to the final velocity for a single stage rocket. (What would R be for a single stage rocket?)

$M_t=1$

$M_p = .1$

$M_2= (1-R) * .9 * M_t$

NASA

$M_1=R * .9 * M_t$

Problem 2 (10 points)

Consider the elliptical orbit of a satellite about the earth. At the point of closest approach, the distance from the center of the earth is 5 earth radii. At the furthest approach, the distance is 20 earth radii. Ignore the moon, although it is out there somewhere. The radius of the earth is $6.37 \times 10^6 m$; the mass of the earth is $5.97 \times 10^{24} kg$; the universal gravitational constant $G = 6.67 \times 10^{-11} m^3/kgsec^2$

a) What is the total period of the satellite about the earth?

b) What is the time of flight from the point of closest approach, to a point located at an angle of 160° as sketched?

c) What is the velocity of the satellite at the point of closest approach. What is the velocity of the satellite at the farthest point.

Problem 3 (10 points)

The Columbia Accident Investigation Board recommended that, for safety reasons, all future shuttle missions should be able to utilize the Space Station as a "safe harbor" in the event of an emergency. This led to the reconsideration of all future Hubble Space Telescope repair missions. Why is it so difficult to get from the Hubble to the Space Station?

To accommodate the participation of the Russians in the space station program–so that they could launch from their Baikonur launch site–the inclination of the space station orbit is 51.6^o from the equator; its altitude is 400 km from the earth's surface.

The inclination of the Hubble orbit–minimizing the energy required for a launch from Kennedy Space Center in Florida– is 28.5^o from the equator and it altitude is 600 km from the earth's surface.

Create an emergency-transfer orbit to transfer the shuttle and its crew from their mission in the Hubble orbit to the space station in the space station orbit. This transfer involves both an angular change and an orbital radius change. Do the angular change first: change orbit inclination without changing orbit altitude.

Cape Canaveral

The International Space station is at 51.6 degrees, the standard launch angle used by Russia at Baikonur, Kazakhstan.

Orbit Angle of space station: 51.6^o from Equator

Baikonur

Equator

Orbit Angle of Hubble: 28.5^o from Equator

It's Rocket Science: Once an object is launched into orbit, changing the angle at which it crosses the Equator is very difficult. The Hubble Space Telescope was launched at 28.5 degrees, the easiest angle possible from Cape Canaveral, Fla. Launching at low angles minimizes the amount of rocket thrust needed to achieve orbit.
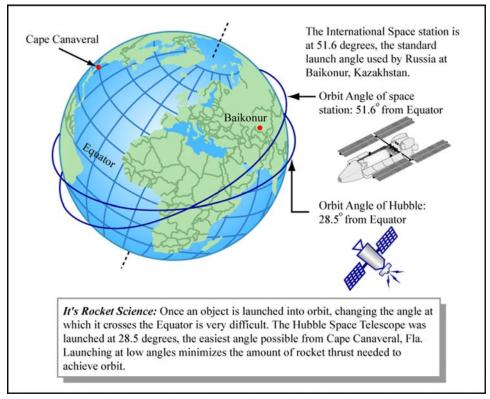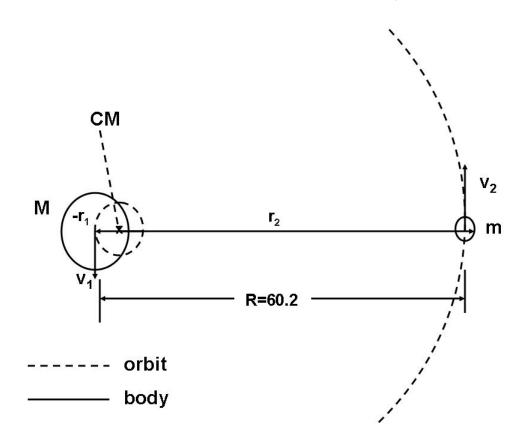
Image by MIT OpenCourseWare.

The two points where the orbit intersects the earth equatorial plane are called the nodes, and the line passing through the nodes is called the line of nodes. Assume that the line of nodes is the same for the Hubble and the Space Station. This means that the angular transfer can be done at the equatorial plane and that the angle of transfer is the difference between the inclination of the two orbits. Recall from the rocket equation that

$$\Delta V = -cln(m_f/m_0) \tag{1}$$

3-1 What percent of the total shuttle mass would need to consist of fuel at this point in its mission to accomplish the required orbit inclination change? Justify the form of the rocket equation used. Take c as 4500 m/sec.

3-2 Now complete the transfer with a Hohmann transfer orbit. Don't worry about time synchronization between the space station and the Hubble at this point. What ΔV and what percentage of shuttle mass are required to perform this maneuver? How does this compare to that required for the inclination change.

3-3 Now considerer time synchronization. How many opportunities per week exist for the total maneuver?

Problem 4 (10 points)

Consider two bodies of mass $M = 5.97 \times 10^{24} kg$ and $m = 7.36 \times 10^{22}$. Non-dimensionalize distances by the radius of the earth ($R_e = 6.36 \times 10^6 m$ and time by $1 day = 24 hours = 3600 * 24 sec$. The masses are separated by a non-dimensional distance of 60.25. They orbit about their common center of mass. Formulate and solve the "Kepler problem" for the orbital motion of each body about their common center of mass. The universal gravitational constant is $G = 6.67 \times 10^{-11} m^3/kgsec^2$.

4-1 Where is the center of mass of the system. How do the bodies move about this center of mass.

4-2 What non-dimensional velocities v_1 and v_2 will produce circular orbits for both bodies?

4-3 What is the non-dimensional radius r_1 of the orbit of M?

4-4 What is the non-dimensional radius r_2 of the orbit of m?

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