NAME : ....

Massachusetts Institute of Technology

16.07 Dynamics

Problem Set 2

Out date: Sept 10, 2007

Due date: Sept 17, 2007

	Time Spent [minutes]
Problem 1	
Problem 2	
Problem 3	
Study Time	

Turn in **each problem** *on* **separate sheets** *so that grading can be done in parallel*

Problem 1 (20 points)

Consider an iceboat traveling in the x direction at constant speed V_{boat} in a true wind of speed V_{wind} making an angle of θ_{wind} with the direction of travel. The forces between the boat and the ice are such that the friction force from the runners is effectively zero while still supporting necessary side forces. In addition, the design of the boat is such that it has a lift to drag ratio, L/D. The helmsman has the freedom to choose the lift to drag ratio within the capabilities of the rig as well as choosing the course relative to the wind direction by choosing θ_{wind}. In this problem, we will explore how best to set these quantities for maximum speed.

Figure 1: Iceboat traveling on ice.
Photograph by tom.lothian on Flickr.
http://www.flickr.com/photos/39948787@N07/4157886943/in/photostream/

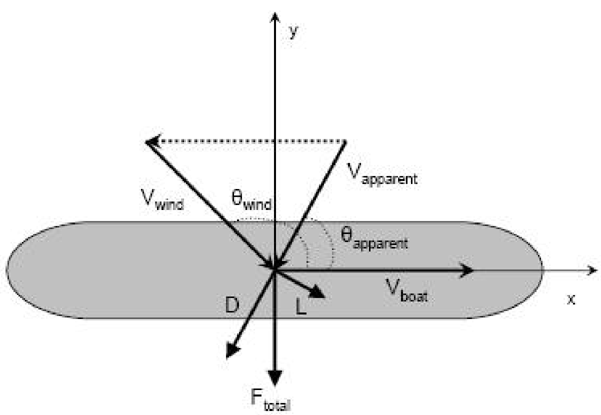


Figure 2: Top view of boat.

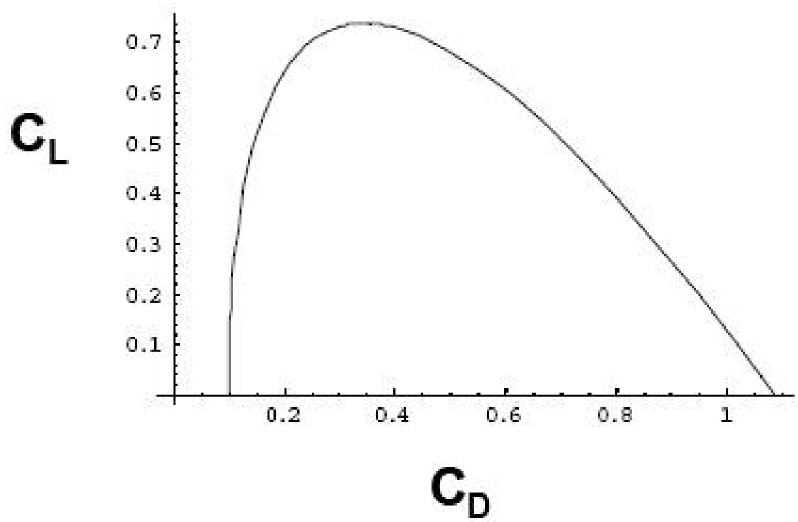
a) Solve for the magnitude of the apparent wind, $V_{apparent}$, seen by the helmsman and its angle relative to the direction of travel, $\theta_{apparent}$.

b) Show that if the boat is traveling at constant speed, the resultant aerodynamic force F_{total} acts in a direction normal to the direction of travel.

c) If the lift to drag ratio is L/D, determine the relation between the speed of the boat, V_{boat}, and the parameters of the true wind – magnitude, V_{wind}, and direction, θ_{wind}. *Recall that D acts along the direction of $V_{apparent}$.* Here you should consider the more relevant case ($0° < \theta_{apparent} < 90°$).

d) For a given L/D, determine what angle θ^*_{wind} should be chosen to achieve maximum boat speed V_{boat}. Determine $V_{boat\,max}$. A nice closed form expression is possible. Argue that the maximum speed is reached if the rig is set to obtain maximum L/D for the rig. *Note that because of symmetry considerations, there will be two such angles – let us consider here $0° < \theta_{wind} < 180°$.*

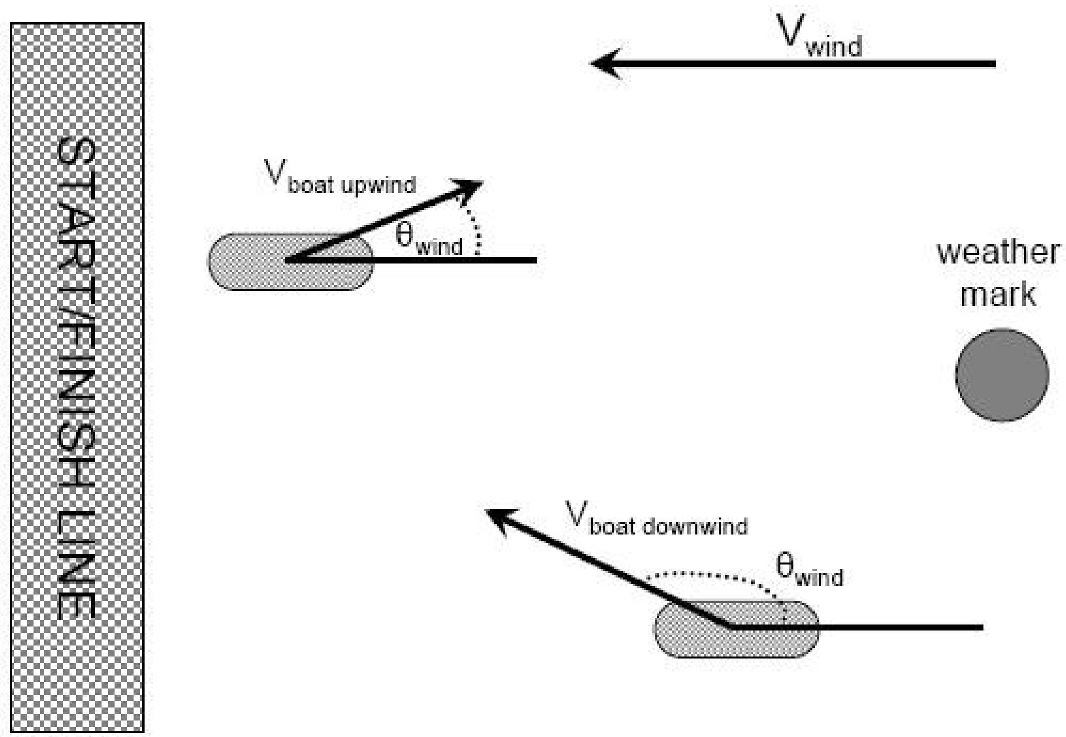
e) A typical plot of lift vs. drag is shown below. This is reasonable for an iceboat. Estimate from this figure the maximum value of L/D, and use this in your further calculations.



For this L/D and a wind speed of 20 knots, what is the maximum speed of the iceboat and what angle should the helmsman make to the true wind direction to achieve this speed? Does this angle have any significance relative to the plot of L/D?

f) Now that we've had our fun, let's race. Race courses are usually set with one mark (the weather mark) directly to windward (aligned and opposite to the wind direction), returning to finish at the starting line. Obviously the boat cannot sail directly into the wind but must sail a course that maximizes the component of the velocity in the wind direction. (And of course the boat can "tack" – change course so that $\theta_{wind} = -\theta_{wind}$.) What is the smallest value of θ_{wind} that can be obtained? What speed will be reached at this angle? Does this angle have any significance relative to the maximum value of L/D? Or to the curve of L vs. D shown above?

Also, although one could sail directly in the wind direction to the finish, this is unlikely to produce the maximum component of speed in the downwind direction. What would be the maximum speed that could be reached sailing directly downwind? ($\theta_{wind} = 180°$)

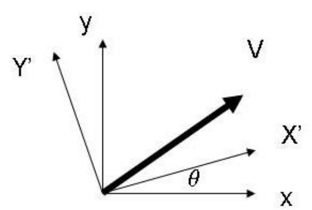


g) In sailing, the term "Velocity Made Good" (VMG) refers to the velocity component directed towards a desired waypoint or destination. VMG is a measure of how well progress towards the waypoint is being made. In the case of our race course, the VMG would be the component of velocity in the direction of the wind.

Using the value of L/D from Part (e), calculate the maximum upwind VMG. What angle, θ_{wind}, should be chosen to achieve this? Repeat the calculations for the downwind leg.

Problem 2 (10 points)

In two-dimensions, a coordinate transformation of the components of a general vector V from an x,y system to an X', Y' system by a rotation through an angle q is given by (in matrix notation)



$$\begin{vmatrix} X' \\ Y' \end{vmatrix} = \begin{vmatrix} \text{Cos } \theta & \text{Sin } \theta \\ -\text{Sin } \theta & \text{Cos } \theta \end{vmatrix} \begin{vmatrix} x \\ y \end{vmatrix}$$

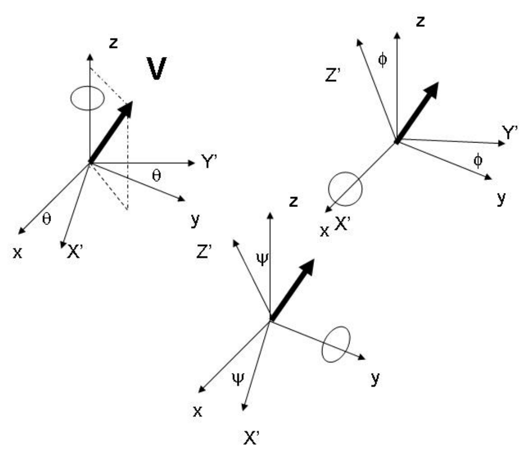
$$\mathbf{X' = [\ A\ ]\ X}$$

Derive the equivalent transformation matrix A in three dimensions for the following different coordinate transformations 1, 2, and 3 shown below.

a) 1. A single rotation about the z axis through an angle θ

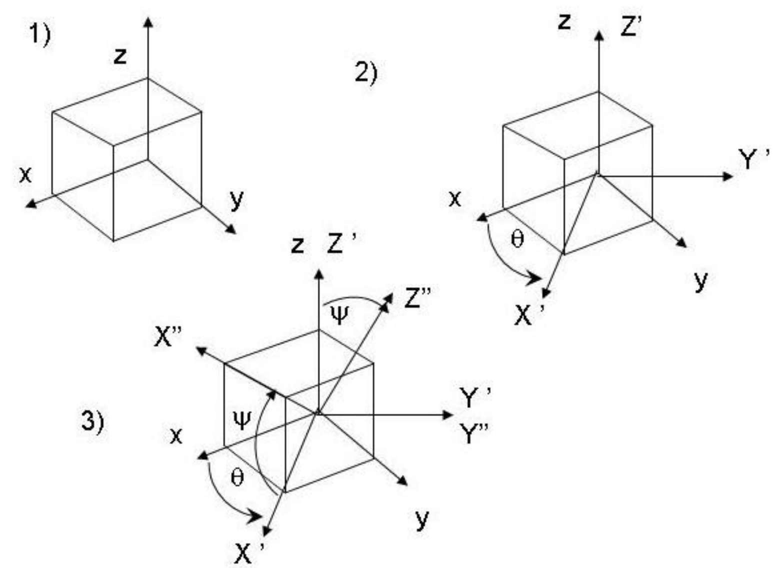
b) 2. A single rotation about the x axis through an angle ϕ

c) 3. A single rotation about the y axis through an angle ψ



Problem 3 (10 points)

Numerical MATLAB problem. Now consider a unit cube located in the x, y, z coordinate system with one edge at the origin. It is desired to perform a coordinate transformation so that the X" axis goes through the center of the cube along the diagonal as shown. (We will use the results of this problem later in the term, so all is not lost. And I am actually going to tell you how to do it, step by step. The steps are not unique, there are many way to accomplish this.)



a) First, perform a coordinate transformation by a rotation q about the z axis. Since we wish to have the X" coordinate line up with the center of the cube, $\theta = 45$ degrees. Relate X' to x through the first transformation matrix.

b) Now perform a rotation about the Y' axis to position the X" axis through the upper corner of the cube. What angle y is required for this rotation? Relate X" to X' through the second transformation matrix.

c) Now relate X" to x through a suitable multiplication of the transformation matrices. Do this step numerically using the matrix manipulation routines of MATLAB. Save this matrix, you will use it again.

d) Check your answer by transforming the vector V=1,1,1 in the x coordinate system to the X" system. What should you get in the X" system?