16.06 Principles of Automatic Control
Recitation 5

Problem 1.

Sketch the positive (180°) and negative (0°) root locus for following system:

$$G(s) = \frac{s-2}{(s^2+1)(s+2)(s+5)}$$

.

Poles are at $s = \pm j$, -2, -5.

Zero are at $s = 2$.

$\alpha = \frac{(0+0-2-5)-(2)}{4-1} = -3$

Asymptotes are at angles of $\pm 60^\circ$, $180^\circ$for positive locus.

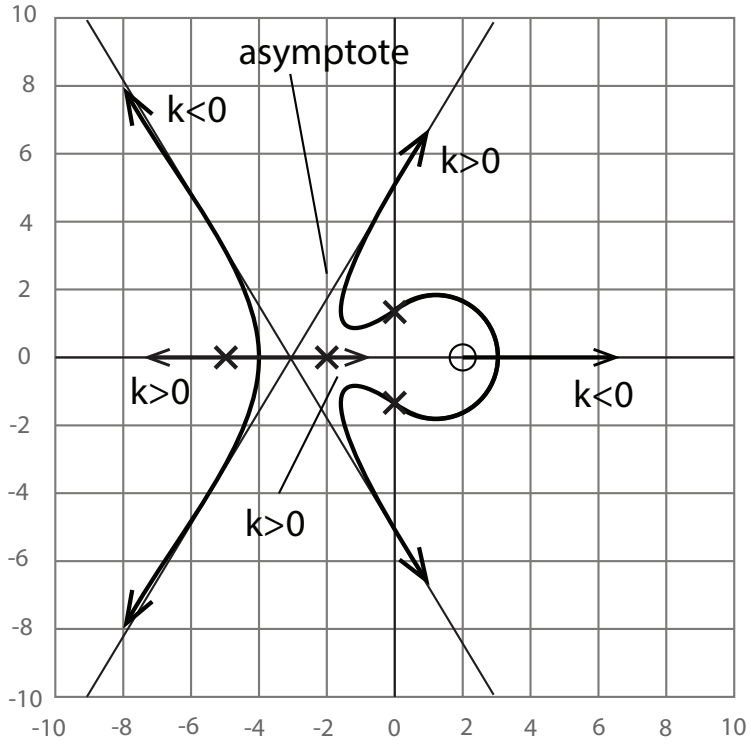
Asymptotes are at angles of $\pm 120^\circ$, $0^\circ$for negative locus.

Angle of departure from pole at $s = +j$ for positive locus is

$$\begin{aligned}\Psi_{\text{dep}} &= \sum \phi - \sum \psi + 180 \\ &= \tan^{-1}\left(\frac{1}{-2}\right) - \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{2}{0}\right) + 180^\circ \\ &= 153.4^\circ - 26.6^\circ - 113^\circ - 90^\circ + 180^\circ = 205.6^\circ\end{aligned}$$

For the negative locus the departure angle is

$$\Psi_{\text{dep}} = \tan^{-1}\left(\frac{1}{-2}\right) - \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{2}{0}\right) = 25.6^\circ$$



Problem 2.

Sketch the negative root locus for the following system:

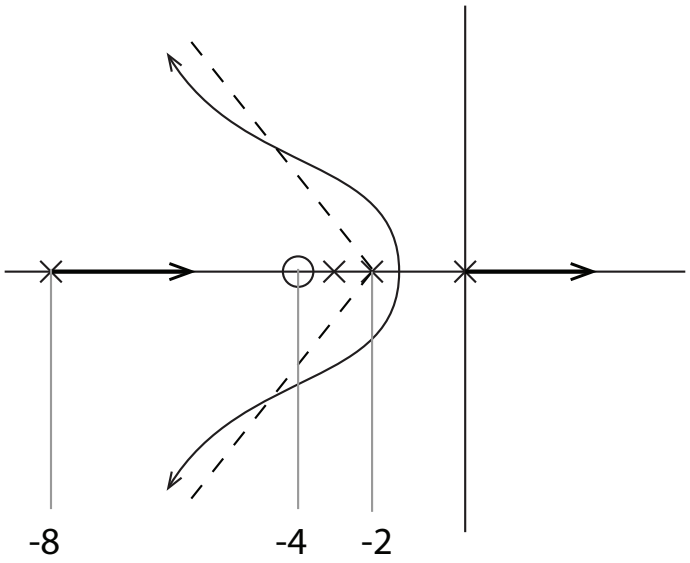
$$G(s) = \frac{s+4}{s(s+2)(s+3)(s+8)}$$

Poles are at $s = 0, -2, -3, -8$.

Zero are at $s = -4$.

$\alpha = \frac{(0-2-3-8)-(-4)}{3} = -3$

Asymptotes are at $0^\circ, \pm 120^\circ$.



The zero attracts the locus, so it actually crosses the asymptotes, which is permissible.

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