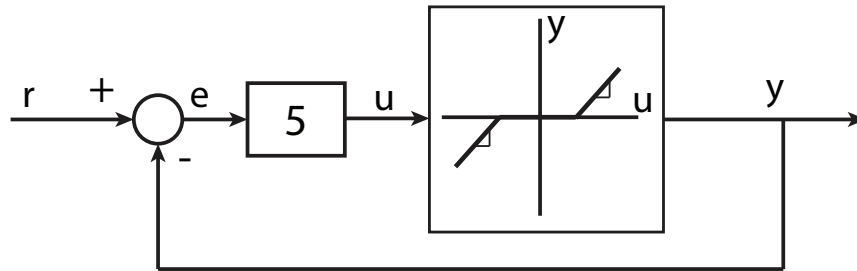
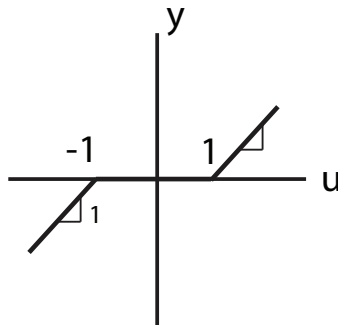


# 16.06 Principles of Automatic Control

## Recitation 1



Find  $y$  as a function of  $r$ :

$$\left. \begin{array}{l} u = 5e \\ e = r - y \end{array} \right\} \text{These come from feedback diagram}$$

Now substitute  $u = 5(r - y)$

$y$  is a piecewise function.

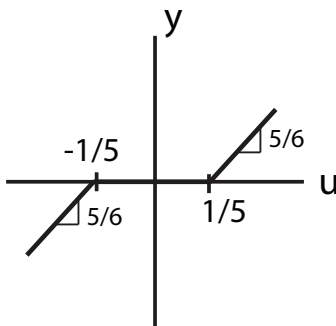
$$y = \begin{cases} u - 1, & u > 1 \\ 0, & -1 < u < 1 \\ u + 1, & u < -1 \end{cases}$$

Now do a substitution for each of the regions:

<b>u &gt; 1</b>	<b>-1 &lt; u &lt; 1</b>	<b>u &lt; -1</b>
$y = 5(r - y) - 1$	$y = 0$	$y = 5(r - y) + 1$
$6y = 5r - 1$		$6y = 5r + 1$
$y = \frac{5}{6}r - \frac{1}{6}$		$y = \frac{5}{6}r + \frac{1}{6}$

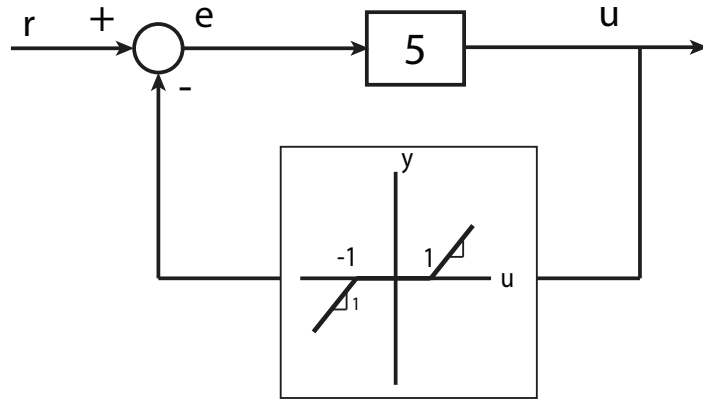
Last step is to find a breakpoint:

<b>u &gt; 1</b>	<b>u &lt; -1</b>
$5(r - y) > 1$	$5(r - y) < -1$
$r - \frac{5}{6}r + \frac{1}{6} > \frac{1}{5}$	$r - \frac{5}{6}r - \frac{1}{6} < -\frac{1}{5}$
$r > \frac{1}{5}$	$r < -\frac{1}{5}$



So the system becomes less non-linear. Increasing the gain would reduce the deadband area and increase the slope towards 1.

Now have non-linear portion in feedback loop.



So now output is  $u$ , and we want to find  $u$  as a function of  $r$ .

$$\left. \begin{array}{l} u = 5e \\ e = r - y \end{array} \right\} \text{equations from block diagram}$$

$$u = 5(r - y) \rightarrow \text{substitution}$$

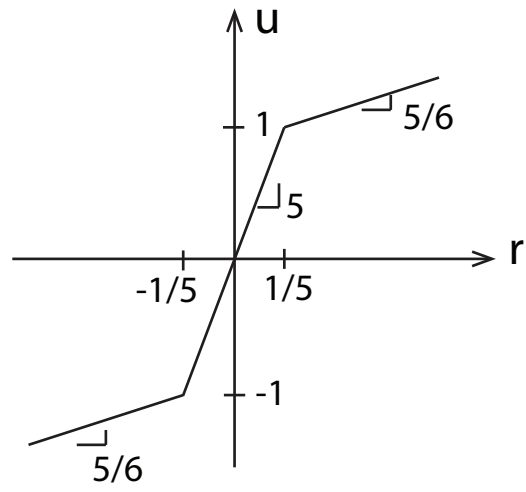
$$y = \begin{cases} u = 5e, u > 1 \\ e = r - y, -1 < u < 1 \rightarrow \text{these are equations of non-linear function} \\ u + 1, u < -1 \end{cases}$$

**Substitute for  $y$  in each piece:**

$u < -1$	$-1 < u < 1$	$u > 1$
$u = 5r - 5u - 5$	$u = 5r$	$u = 5r - ru + 5$
$6y = 5r - 5$		$6y = 5r + 5$
$u = \frac{5}{6}r - \frac{5}{6}$		$u = \frac{5}{6}r + \frac{5}{6}$

For limits on  $r$ :

$u < -1$	$u > 1$
$\frac{5}{6}r - \frac{5}{6} < -1$	$\frac{5}{6}r + \frac{5}{6} > 1$
$r < -\frac{1}{5}$	$r > \frac{1}{5}$



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