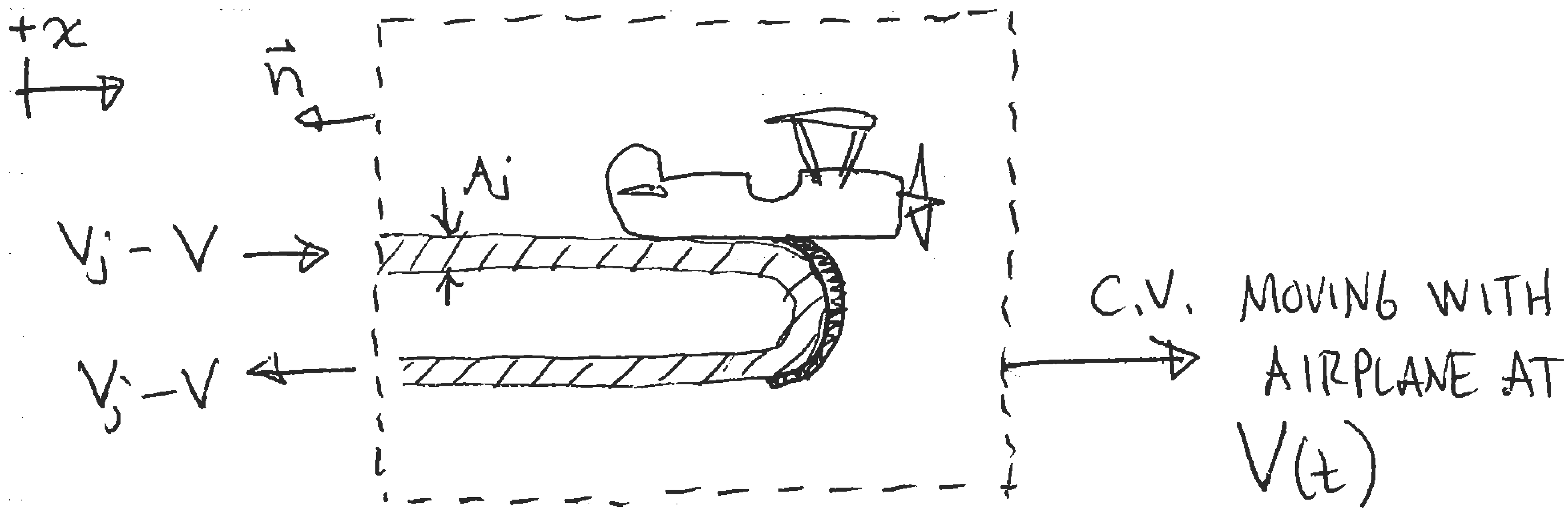
UNIFIED PROPULSION P2 SOLUTIONS (WAITZ)

USE INTEGRAL MOMENTUM THEOREM IN VEHICLE REFERENCE FRAME.
VELOCITY OF REFERENCE FRAME = V (VELOCITY OF VEHICLE).
ALL OTHER VELOCITIES ARE RELATIVE TO V AND ARE LABELED $\vec{u}$.

a)
$$\underbrace{\sum F_x}_{\substack{\text{SUM OF EXTERNAL FORCES = 0 SINCE}\\ \text{PROBLEM SAYS TO NEGLECT DRAG}}} = \underbrace{\frac{dV}{dt}\int_{V_{ol}} \rho \, dV_{ol}}_{\substack{\text{ACCELERATION RELATIVE TO INERTIAL REF. FRAME}\\ = M\frac{dV}{dt}}} + \underbrace{\int_{V_{ol}} \frac{d(\rho u_x)}{dt} dV_{ol}}_{\substack{\text{CHANGE IN MOM. OF MASS WITHIN C.V.}\\ \text{RELATIVE TO C.V. REF. FRAME}\\ = 0 \text{ SINCE MASS OF VEHICLE NOT CHANGING AND}\\ \text{CAN NEGLECT ANY UNSTEADY CHANGES IN MOMENTUM OF}\\ \text{WATER PER PROBLEM STATEMENT}}} + \underbrace{\int_S u_x(\rho\vec{u})\cdot\vec{n}\, dS}_{\substack{\text{NET FLUX OF MOM. ACROSS}\\ \text{BOUNDARIES OF C.V.}}}$$



$$0 = M\frac{dV}{dt} + \int_S u_x(\rho\vec{u})\cdot\vec{n}\, dS$$

b) $$-M\frac{dV}{dt} = \underbrace{\overbrace{-A_j\rho_j\,(V_j-V)(V_j-V)}^{\text{flow in}}}_{\substack{\text{OPPOSITE} \\ \text{TO NORMAL}}}\;\underbrace{\overbrace{-A_j\rho_j\,(V_j-V)(V_j-V)}^{\text{flow out}}}_{\substack{\text{NEGATIVE} \\ \text{X-DIRECTION}}}$$

NOTE THE SIGNS ARE BOTH NEGATIVE BUT FOR DIFFERENT REASONS!

EQNS OF MOTION:

$$\boxed{\begin{array}{ll} \dfrac{dV}{dt} = \dfrac{2A_j\rho_j}{M}(V_j-V)^2 & V_j > V \\[2ex] \dfrac{dV}{dt} = 0 & V_j < V \end{array}}$$

c) $$F = -2\rho_j A_j (V_j-V)^2$$

$$T = -F = 2\rho_j A_j (V_j-V)^2$$