Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

16.01/16.02 Unified Engineering I, II
Fall 2003

Systems Problem 4

Name: _____

Due Date: Parts 1, 2, 3 - 10/2/2003
Parts 4, 5, 6, 7 - 10/9/2003

	Time Spent (min)
Part 1	
Part 2	
Part 3	
Part 4	
Part 5	
Part 6	
Part 7	
Study Time	

Announcements: Please turn in this cover sheet on 10/9/2003

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Aeronautics and Astronautics
16.010 / 16.020 Unified Engineering

Systems Problems #4

Issued: September 25, 2003   Part 1, 2, and 3 Due: 5:00p October 2, 2003
                                           Parts 4, 5, 6, and 7 Due: 5:00p October 9 2003

WATER BOTTLE ROCKET PERFORMANCE ANALYSIS

Objectives

After completing this systems problem, you should be able to:

• Apply rocket performance equations and thermodynamics such that you can call yourself a
Novice Rocket Scientist.

• Apply concepts from freshman year (especially 8.01) and from Unified to model a system of
modest complexity.

• Use simple computational tools such as a spreadsheet to compute the performance of a system
based on an engineering model.

• Explain an engineering model, its underlying concepts, its assumptions, and its limitations.

• Use an engineering model for parameter design to improve performance.

Discussion

This is the first of three consecutive systems problems devoted to the design of a small rocket
propelled by compressed air and water. In this systems problem, you will estimate rocket
performance and make a preliminary analysis of the impact of two design parameters on the
maximum altitude achieved by the rocket. This includes developing a computer spreadsheet to
aid in the analysis. The second systems problem (next week) will focus on your design of a new,
higher-performance rocket, while the third systems problem will involve construction of your
rocket, flight testing, and final performance analysis.

We will begin with a baseline rocket system that uses a standard 2 liter soda bottle for the rocket structure and fuel storage. The bottle will be partially filled with water and mounted on a rocket launch mechanism (Figure 1). The air in the bottle will then be pressurized, and the launch will be executed. We will focus on this baseline rocket in this systems problem and predict its performance. In the following systems problem, you will design your own rocket and will perform a performance analysis of your design. So, your work here lays the groundwork for the next several weeks.
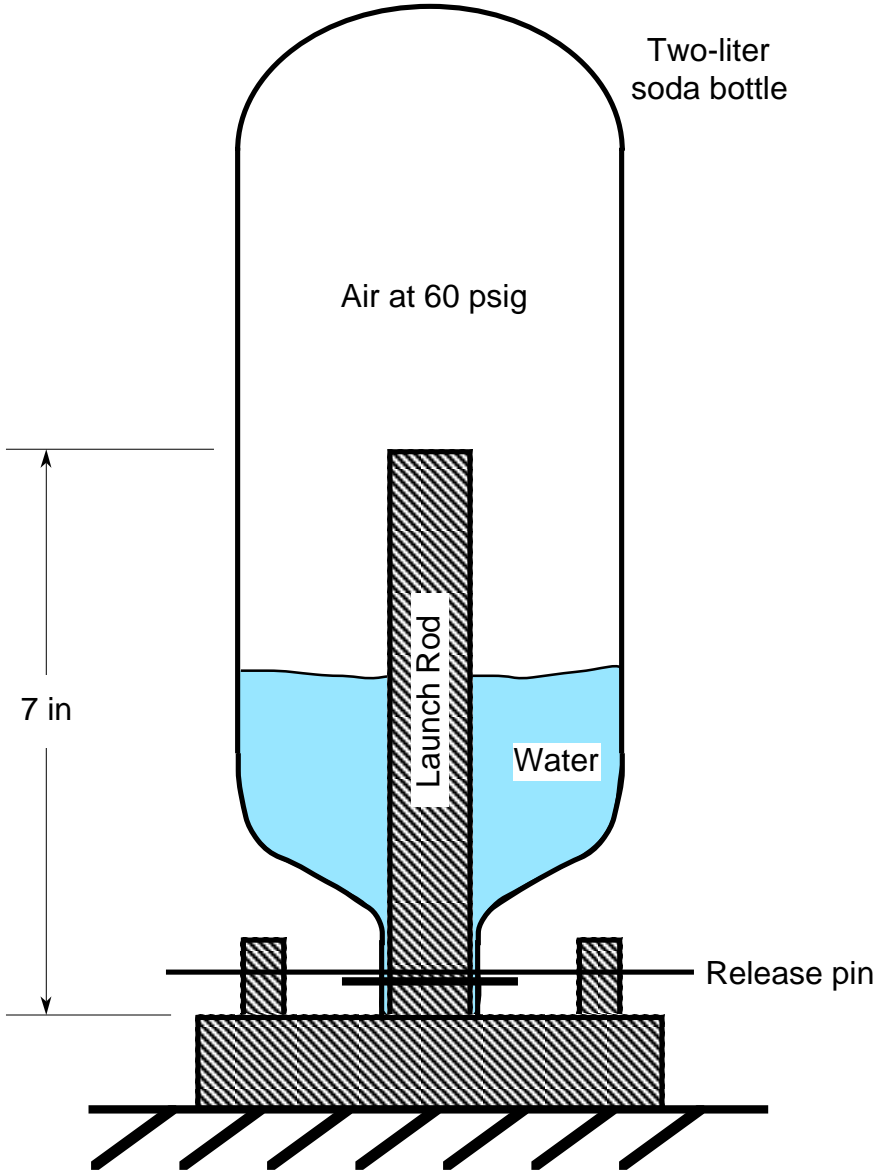

Figure 1.
Water Bottle Rocket and Launching Mechanism

The same launching mechanism will be used for the baseline rocket we examine here and for the rocket you design later. The mechanism uses a 7-inch-long rod to provide stability and to ensure that the rocket proceeds in a straight line. The bottle is inverted onto the rod, and the launcher is staked to the ground. A metal launch restraining pin is inserted over the lip of the bottle neck to keep the bottle on the launcher until it is ready for blastoff. Air is then pumped through the rod to pressurize the bottle; a small rubber o-ring on the launch rod provides an airtight seal at the mouth of the bottle.

The rocket launch consists of three stages. The first stage covers the period between the start of the launch until the rocket nozzle (bottle mouth) just reaches the top end of the launch rod. The second stage involves the compressed air forcing the water out of the bottle at high speed as the bottle rises into the sky. We will assume that any compressed air that remains after the water is expelled produces negligible additional thrust. The third stage is a ballistic stage, in which the rocket continues upward under only the influence of gravity and drag, reaching some maximum altitude before falling back to the ground.
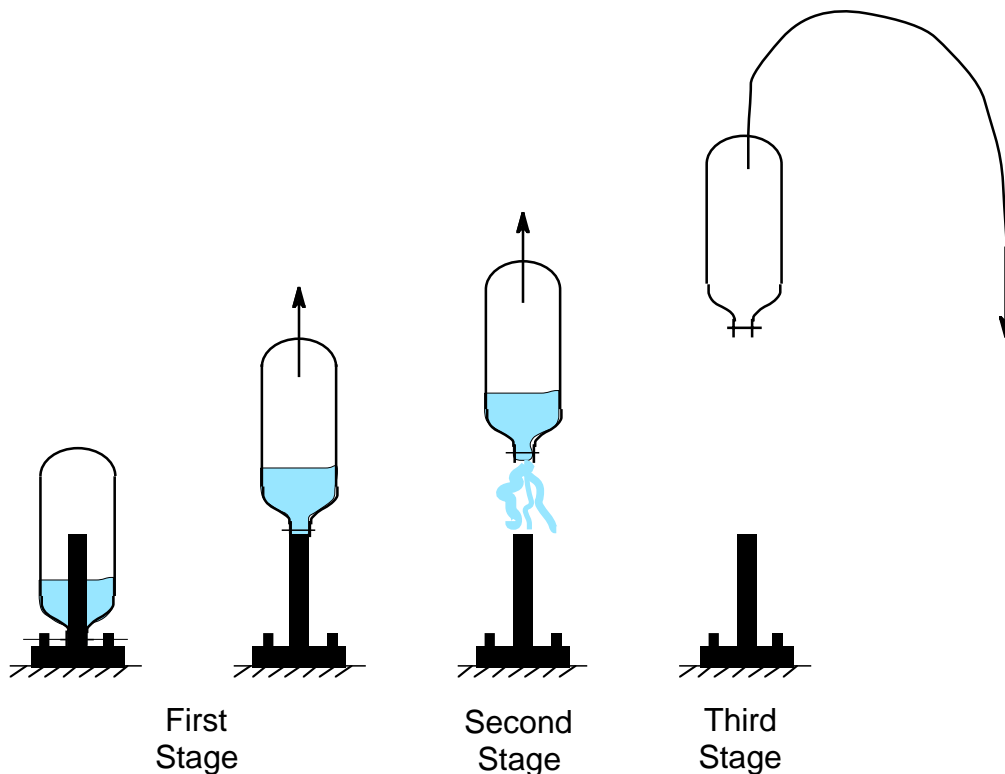

Figure 2.
Rocket Launch Stages

Relevant parameters

Rocket

Volume of bottle	2.3×10^{-3}	m^3
Outer diameter	1.118×10^{-1}	m
Nozzle diameter	2.159×10^{-2}	m
Empty mass	4.7×10^{-2}	kg
Coefficient of drag	0.5 (estimated)	

Launcher

Launch rod length	1.778×10^{-1}	m
Launch rod diameter	2.159×10^{-2}	m

Environmental Conditions

Air pressure	1.013×10^5	Pa
Air temperature	288	K
Air density	1.225	kg/m^3
Water density	1×10^3	kg/m^3

Baseline Launch Procedure
1. Bottle filled 40% of volume with water
2. Bottle inserted onto launch stand
3. Launch stand is securely staked to the ground and launch retainer pin is inserted
4. Bottle is slowly pressurized to 60 psig (60 pounds per square inch gauge pressure = 60 psi above ambient air pressure = 4.137×10^5 Pa above ambient air pressure)

1. Thermodynamic Analysis

The rocket launch process actually forms a complete thermodynamic cycle, composed of the following states:

State 0: air at ambient pressure and temperature
State 1: air compressed in the bottle, at ambient temperature, just before launch
State 2: air compressed in the bottle, at the start of the second stage
State 3: air at atmospheric pressure immediately following the second stage

The transitions between states are:

State 0 to State 1: air is compressed into the bottle isothermally

State 1 to State 2: air expands adiabatically as the launch rod leaves the bottle (first stage)
State 2 to State 3: air expands adiabatically to atmospheric pressure (second stage)
State 3 to State 0: air warms to 288K at atmospheric pressure, completing the cycle

(a) Determine the complete state of the air (p, V, T) at each state in the cycle (0, 1, 2, and 3).

(b) Compute the work W, heat flow Q, and change in internal energy ΔE in each of the four transitions between states.

(c) Verify that the work performed over the entire cycle equals the heat flow, and that the change in internal energy after a complete cycle is zero. (note: you may not find that these hold true exactly, due to roundoff error and to the assumption of constant C_p [specific heat actually varies slightly with temperature] -- getting answers to within a few Joules is fine).

(d) Draw a P-V plot and a T-V plot for this cycle (to scale, not just sketched), allowing one to clearly read off the values of p, V, and T at each state.

2. First Stage

During the first stage, you may neglect friction forces due to the launch rod and you may neglect aerodynamic drag forces (the velocity of the bottle is still relatively small at this point). Do not neglect the weight of the bottle and water, however.

You may assume that the acceleration of the rocket is constant during the entire first stage and is equal to the acceleration at the instant that the launch pin is removed.

(a) Compute the acceleration of the bottle at the instant that the launch pin is removed.

(b) Compute the time at which the bottle nozzle reaches the top end of the launch rod (end of the first stage).

(c) Compute the velocity of the bottle at the end of the first stage.

(d) Compute the work done by the system. You can do this by adding the change in potential energy, kinetic energy, and the work that the compressed air does pushing against the atmosphere during the expansion in the first stage.

(e) Compare the work done by the system from 2(d) against your answer to 1(b) for the first stage.

3. Software Tool Design

You will need to conceive, design, implement and operate (CDIO) a software based analysis tool for parts 4 through 7. Read through parts 4 through 7 and conceive and design (CD) your analysis tool. Document your tool's design through commented pseudo-code. Submit your documented pseudo-code in a neat and readable professional manner.

4. Second Stage

During the second stage, the bottle will expel all of the water and reach a relatively high velocity. Due to the high speed, it is now necessary to include aerodynamic drag as a force. A direct solution of the rocket's state as a function of time is difficult since the bottle's weight, thrust, and drag are constantly changing. You will need to develop a software based analysis tool in Ada to solve this problem by computing the forces, acceleration, velocity, and height in a series of small time steps. Time steps of 0.001 or 0.002 seconds should be appropriate due to the rapid changes that are occurring during the second stage.

When putting together the Ada software tool, it will help you later if you leave parameters such as empty bottle mass and the amount of water in the bottle at start as separate variables that you can change, rather than "hard coding" them into your tool.

(a) Compute the acceleration of the bottle at the start of the second stage, assuming the bottle is just clear of the launch rod and water is free to flow out the nozzle. You may neglect aerodynamic drag for this computation.

(b) Develop a set of equations that will enable you to solve for the bottle's velocity and altitude over time. Assume that at each time step you start with a known mass of water still in the bottle, a known bottle velocity, and a known bottle altitude. From this information, write down the series of equations that will then generate the internal gas volume and pressure, velocity of the water through the nozzle, water mass flow rate, thrust, drag, bottle weight, and acceleration at that time instant. Then write down the difference equations that will allow you to compute the next time step's mass of water in the bottle, next time step's bottle velocity, and next time step's bottle altitude. Leave all of these equations in terms of variables — do not plug in any numerical values. Also clearly list and define each variable that you use.

(c) Using a spreadsheet based on your equations in 3(b), compute the velocity of the bottle when all the water has been expelled. As the starting condition, use the values of water mass, bottle velocity, and bottle altitude that you computed for the end of the first stage.

(d) Compute the altitude of the bottle when all of the water has been expelled.

5. Ballistic Stage

To examine the ballistic stage, you will continue to need your Ada software tool since the bottle's drag force varies greatly as the bottle slows its ascent, and then begins to drop to earth. You will still need a small time step until the bottle slows somewhat since the bottle's deceleration will initially be quite large due to its high speed and correspondingly high drag force. Once the drag force gets smaller, you may want to increase the time step to around 0.05 seconds so that you can observe the rest of the flight without requiring too many computations in the tool.

Be very careful about the sign of the drag force after the bottle reaches its maximum altitude, making sure that it always opposes the velocity vector of the bottle.

As the starting condition, use the velocity and altitude you computed for the end of the second stage in 3(c) and 3(d). Assume also that the air remaining in the bottle is at atmospheric pressure and temperature, and there is no longer any thrust force.

(a) Compute the maximum altitude that the bottle reaches.

(b) Determine the time at which the bottle will reach its maximum altitude.

(c) Determine the total flight time of the bottle, from launch until ground impact.

6. Performance Analysis

Now, we will do a preliminary performance analysis to observe how two design parameters affect maximum altitude. By changing the values of parameters in your software tool, determine the following effects:

(a) Examine how the percentage of water filling the bottle at the start affects maximum altitude. Given an empty bottle mass of 0.047 kg, vary the amount of water in the bottle before launch from 20% to 70% in 10% increments, and write down the maximum altitude attained in each

condition. Explain why the rocket's performance varies as it does with the amount of water used.

(b) Assuming the bottle is filled 40% with water, vary the empty mass of the bottle (this could be done, for example, by adding clay to the bottle) to observe the impact on maximum altitude. Use empty bottle masses of 0.047 kg, 0.15 kg, 0.25 kg, 0.45 kg, 0.65 kg, and 1.2 kg and write down the resulting maximum altitude attained. Explain why the rocket's performance varies as it does with extra mass.

7. Model Assumptions and Limitations

Discuss the limitations of your model that was used to examine the performance of the water bottle rocket. What simplifying assumptions were made that could affect the estimate of performance? What modifications to the model could be made to improve it?