Lecture F9 Mud: Momentum Theorem Applications
(30 respondents)

1. **What does $\oint -p\hat{n}\, dA$ physically represent?** (1 student)
   The net force that the pressure distribution applies to the control volume.

2. **Why is momentum flow added to the pressure force to get the total force?**
   (3 students)
   I wouldn't phrase it that way. One way to write the governing relation is:

   $$\oint -p\hat{n}\, dA \;-\; \vec{R}' \;=\; \oint \rho\vec{V}\cdot\hat{n}\,\vec{V}\, dA$$

   Or in words: The pressure force ($\oint -p\hat{n}\, dA$) plus internal body force ($-\vec{R}'$) that are applied to the volume are balanced by a net momentum flow out of the volume ($\oint \rho\vec{V}\cdot\hat{n}\,\vec{V}\, dA$).

3. **What is $-p\hat{n}\cdot\hat{\imath}$?** (1 student)
   $-p\hat{n}$ is the local pressure force on the control volume boundary.
   $-p\hat{n}\cdot\hat{\imath}$ is just the x-component of this force.

4. **Explain the wing-above-ground example better?** (3 students)
   If you put the left, right, and top control volume boundaries far from the wing, only the pressure along the bottom boundary contributes to the momentum integral (in mud point 2). We set $\vec{R}' = L'\hat{\jmath}$ as given. Along the bottom boundary, we also have $\hat{n} = -\hat{\jmath}$. The integral boils down to

   $$\int_{\text{bottom}} -(p - p_\infty)\,\hat{\jmath}\, dA \;=\; -L\hat{\jmath}$$

   The lefthand side is precisely the force that the overpressure imparts to the ground, and it's equal to $-L\hat{\jmath}$. Intuitively, the wing's downward push inside the volume must be balanced by the ground's upward push on the bottom boundary, since there is no net momentum flow (see mud point 2 above). Since the ground is pushing up on the air with a force of L', the air must be pushing down on the ground with a force of L'.

5. **Math too dense. Too many variables. Too much too fast.** (5 students)
   I realize all the integrals are intimidating. You should put most of your effort at grasping the underlying physical concepts (e.g. mud point 2 above). The integrals are just expressions which have to be evaluated to get numerical answers out of the physical principle. Doing the homework problems is good practice for this. See me or the grad TA's if you need help.

6. **No mud** (13 students)