1. $$G(j\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \delta(t-T)\, e^{-j\omega t}\, dt$$

$$= e^{-j\omega T} \quad \text{(Using the "sifting property")}$$

$\therefore$,

$$\boxed{G(j\omega) = e^{-j\omega T}}$$

2. $$G(j\omega) = \int_{-T}^{T} 1 \cdot e^{-j\omega t}\, dt$$

$$= \frac{-1}{j\omega} e^{-j\omega t} \Big|_{t=-T}^{T}$$

$$= \frac{-1}{j\omega}\left[e^{-j\omega T} - e^{+j\omega T}\right]$$

$$= \frac{1}{j\omega}\left[e^{+j\omega T} - e^{-j\omega T}\right]$$

$G(j\omega)$ can be simplified by application of Euler's formula, or by inspection. The result is

$$\boxed{G(j\omega) = \frac{2}{\omega} \sin \omega T}$$

3. $$G(j\omega) = \int_{-\infty}^{\infty} \frac{1}{t^2+T^2} e^{-j\omega t}\, dt$$

<u>But</u>, I don't know how to do this integral.
Use duality:

If $\mathcal{F}[g(t)] = f(\omega)$, then

$$\mathcal{F}[f(t)] = 2\pi\, g(-\omega)$$

$g(-\omega)$ is given by

$$g(-\omega) = \frac{1}{(-\omega)^2 + T^2} = \frac{1}{\omega^2+T^2}$$

$$= \frac{1}{-s^2+T^2} = \frac{-1}{(s+T)(s-T)}$$

$$= \frac{1/2T}{s+T} - \frac{1/2T}{s-T}$$

$$= \frac{1}{2T}\left[\frac{1}{j\omega+T} - \frac{1}{j\omega-T}\right]$$

Therefore,

$$f(t) = 2\pi\, \mathcal{F}^{-1}\left[g(-\omega)\right]$$

$$= 2\pi \frac{1}{2T}\left[e^{-tT}\sigma(t) + e^{+tT}\sigma(-t)\right]$$

$$= \frac{\pi}{T} e^{-|t|T}$$

$\therefore$

$$\boxed{G(j\omega) = f(\omega) = \frac{\pi}{T} e^{-|\omega| T}}$$

4. $$g(t) = \frac{\sin \pi t/T}{\pi t/T}$$

Use duality:

$$\mathcal{F}[g(t)] = G(j\omega) = f(\omega)$$

$$\mathcal{F}[f(t)] = 2\pi\, g(-\omega)$$

In this case,

$$g(-\omega) = \frac{\sin(-\pi\omega/T)}{-\pi\omega/T} = \frac{\sin \pi\omega/T}{\pi\omega/T}$$

If we let $T' = \pi/T$, this becomes

$$g(-\omega) = \frac{\sin \omega T'}{\omega T'}$$

The inverse FT (From part 1) is

$$\mathcal{F}^{-1}[g(-\omega)] = \mathcal{F}^{-1}\left(\frac{\sin \omega T'}{\omega T'}\right)$$

$$= \frac{1}{2T'} \mathcal{F}^{-1}\left(\frac{\sin \omega T'}{\omega/2}\right)$$

$$= \begin{cases} 1/2T', & |t| \le T' \\ 0, & \text{else} \end{cases}$$

$$= f(t)/2\pi$$

Therefore,

$$f(t) = \begin{cases} \pi/T', & |t| \le T' \\ 0, & \text{else} \end{cases}$$

$$\Rightarrow \quad G(j\omega) = f(\omega)$$

$$= \begin{cases} \pi/T', & |\omega| \le T' \\ 0, & \text{else} \end{cases}$$

$$= \begin{cases} T, & |\omega| \le \pi/T \\ 0, & \text{else} \end{cases}$$

5. Let $\quad F(j\omega) = \dfrac{\sin \omega T}{\omega T}$

Then $\quad G(j\omega) = [F(j\omega)]^2$

$\Rightarrow \quad g(t) = f(t) * f(t)$ (convolution property)

Using the results of part (1),

$$f(t) = \mathcal{F}^{-1}\left[\frac{\sin\omega T}{\omega T}\right]$$

$$= \frac{1}{2T}\,\mathcal{F}^{-1}\left[\frac{\sin\omega T}{\omega/2}\right]$$

$$= \begin{cases} 1/2T, & |t| \le T \\ 0, & \text{else} \end{cases}$$

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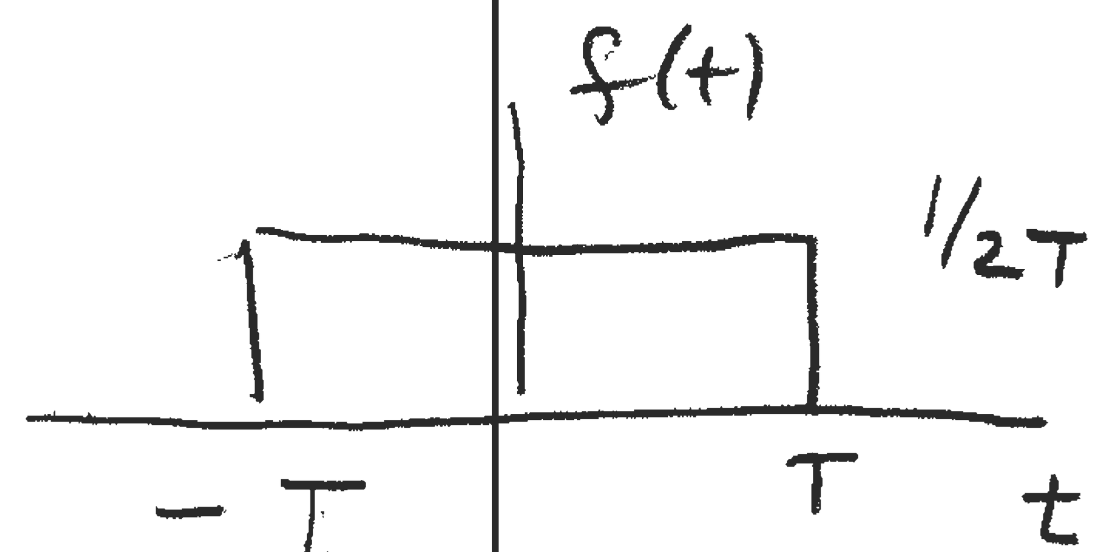

g(t) is the convolution of f(t) with f(t), which is