1. The aircraft is flying at 120 knots. Therefore,

$$V_0 = 120 \text{ kn} \times \frac{6080 \text{ ft}}{\text{NM}} \times \frac{1}{3600 \text{ s/hr}} \times 0.3048 \frac{\text{m}}{\text{ft}}$$

$$= 61.77 \text{ m/s}$$

Also,

$$g = 9.82 \text{ m/s}^2, \quad L_0/D_0 = 10$$

Therefore, the matrix A is given by

$$A = \begin{bmatrix} 0 & 0 & 61.77 \\ 0 & -0.03180 & -9.82 \\ 0 & 0.005147 & 0 \end{bmatrix}$$

The eigenvalues are the roots of

$$\det(sI - A) = 0$$

$$= s\left[(s + 0.0318)s + (0.005147)(9.82)\right]$$

$$= s(s^2 + 0.03180 s + 0.05055)$$

The roots can be found using the quadratics formula. So

$$s_1 = 0, \quad s_2 = -0.01590 + 0.2243j$$

$$s_3 = -0.01590 - 0.2243j$$

The eigenvectors are found by solving

$$(s_i I - A)\,\underline{x}_i = \underline{0}$$

Do each in turn:

<u>$s_1 = 0$</u>:

$$s_1 I - A = \begin{bmatrix} 0 & 0 & -61.77 \\ 0 & +0.03180 & +9.82 \\ 0 & -0.005147 & 0 \end{bmatrix}$$

Since the 1st column is all zeros, a solution is

$$\underline{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

<u>$s_2 = -0.01590 + 0.2243j$</u>:

$s_2 I - A =$

$$\begin{bmatrix} -0.01590 + 0.2243j & 0 & -61.77 \\ 0 & 0.01590 + 0.2243j & 9.82 \\ 0 & -0.005147 & -0.1590 + 0.2243j \end{bmatrix}$$

Row reduction proceeds as normal, but is messy. The result is

$$\begin{bmatrix} 1 & 0 & -19.43 - 274.1j \\ 0 & 1 & 3.0885 - 43.57j \\ 0 & 0 & 0 \end{bmatrix}$$

Note that one row is zero, as it should be, if $\det(s_2 I - A) = 0$!

Arbitrarily take 3rd element of $\underline{X}_2 = 1$.

The

$$\underline{X}_2 = \begin{bmatrix} 19.43 + 274.1j \\ -3.0885 + 43.57j \\ 1 \end{bmatrix}$$

$s_3 = -0.0159 - 0.2243j$:

Because $s_3 = s_2^*$ (complex conjugate),

$$\underline{X}_3 = \underline{X}_2^* = \begin{bmatrix} 19.43 - 274.1j \\ -3.0885 - 43.57j \\ 1 \end{bmatrix}$$

2. The general solution is

$$\underline{x}(t) = a_1 \underline{X}_1 e^{s_1 t} + a_2 \underline{X}_2 e^{s_2 t} + a_3 \underline{X}_3 e^{s_3 t}$$

The initial condition is

$$\underline{x}(0) = a_1 \underline{X}_1 + a_2 \underline{X}_2 + a_3 \underline{X}_3$$

$$= [\underline{X}_1 \ \underline{X}_2 \ \underline{X}_3]\, \underline{a} \equiv V \underline{a}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix}$$

Therefore,

$$\underline{a} = \begin{bmatrix} 3.885 + 0j \\ 0.05 - 0.003544j \\ 0.05 + 0.003544j \end{bmatrix}$$

I found this solution using Matlab, but it could easily be done with a calculator.

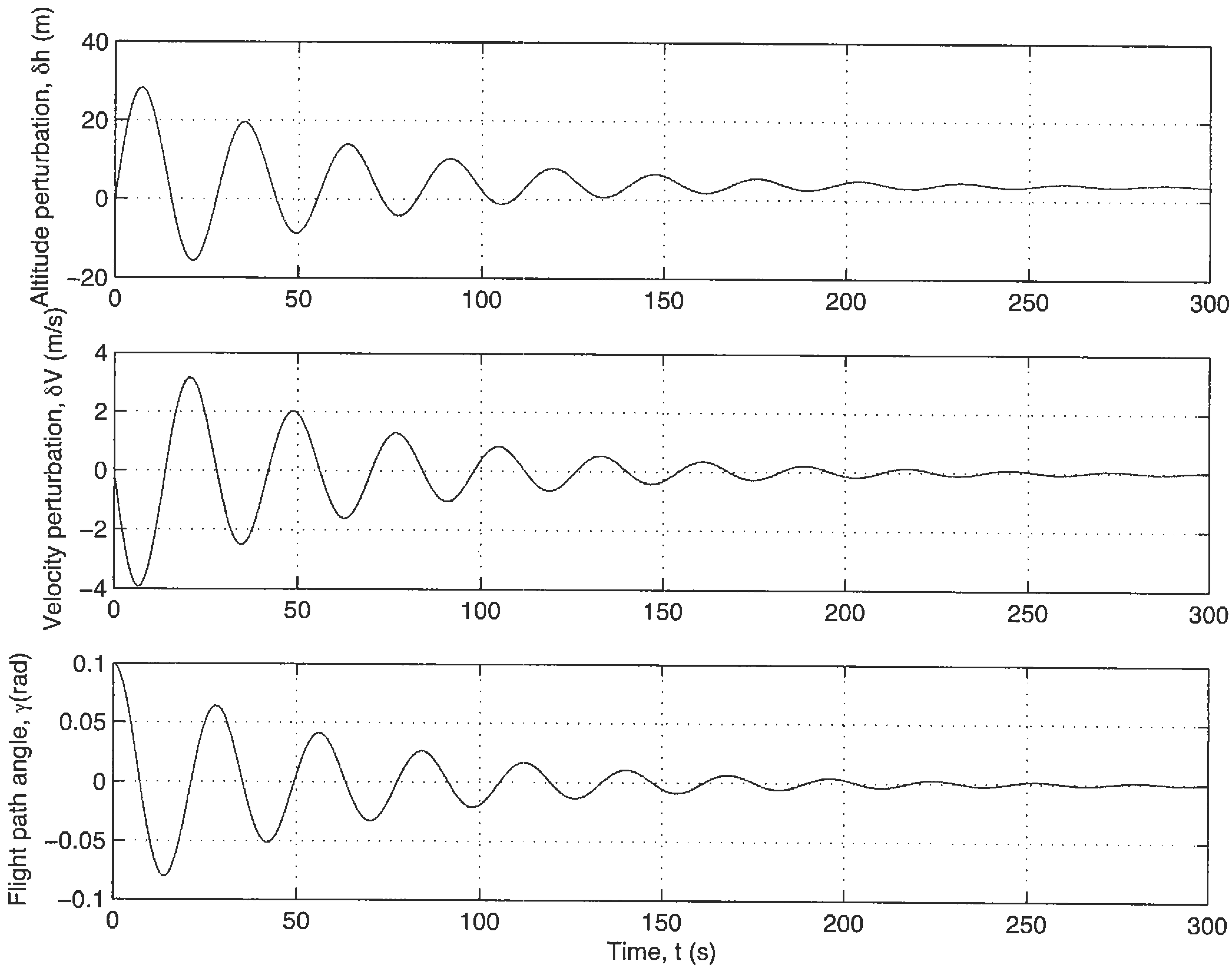
The result can now be plotted, since Matlab does complex exponentials.

```
s1 = 0;
s2 = -0.0159 + 0.2243j;
s3 = -0.0159 - 0.2243j;
X1 = [1;0;0];
X2 = [-19.43-274.1j;-3.0885+43.57j;1];
X3 = [-19.43+274.1j;-3.0885-43.57j;1];
a1 = 3.885;
a2 = 0.05-0.003544j;

a3 = 0.05+0.003544j;
t = 0:0.5:300;
x = a1*X1*exp(s1*t)+a2*X2*exp(s2*t)+a3*X3*exp(s3*t);

subplot(311)
plot(t,real(x(1,:)))
ylabel('Altitude perturbation, \delta{}h (m)')
grid
subplot(312)
plot(t,real(x(2,:)))
ylabel('Velocity perturbation, \delta{}V (m/s)')
grid
subplot(313)
plot(t,real(x(3,:)))
ylabel('Flight path angle, \gamma{}(rad)')
grid
xlabel('Time, t (s)')

print -depsc phugoid.eps
```


Altitude perturbation, δh (m)
40
20
0
−20
Velocity perturbation, δV (m/s)
4
2
0
−2
−4
Flight path angle, γ(rad)
0.1
0.05
0
−0.05
−0.1
0
50
100
150
200
250
300
Time, t (s)