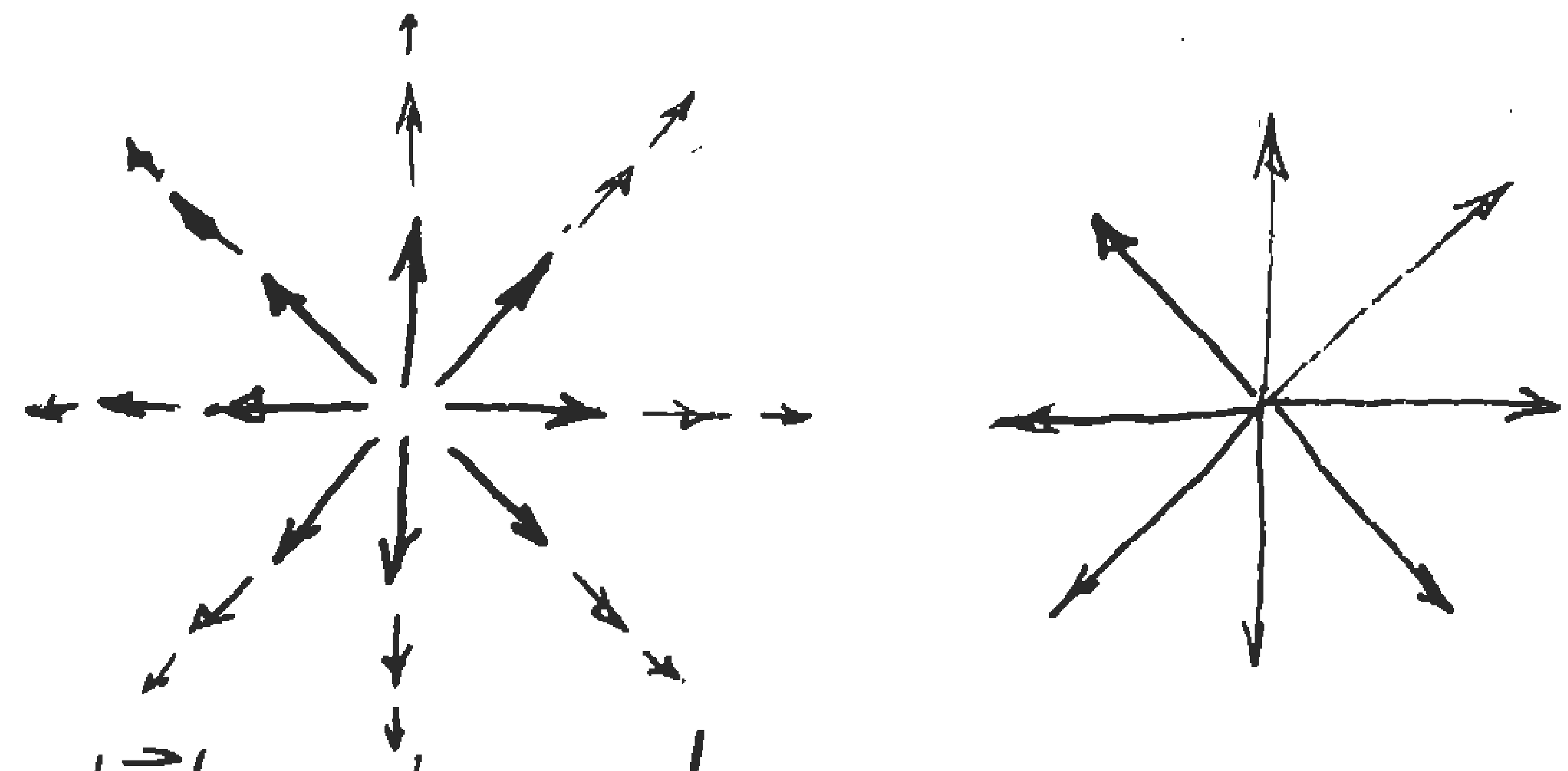
UE Fluids | F12 Solution | Fall '03

a) $\psi = \arctan\left(\frac{y}{x}\right)$

$$u = \frac{\partial \psi}{\partial y} = \frac{x}{x^2+y^2}$$

$$v = -\frac{\partial \psi}{\partial x} = \frac{y}{x^2+y^2}$$

Streamlines: $\arctan\left(\frac{y}{x}\right) = \theta = \text{const}$



$|\vec{V}|$ varies as $\frac{1}{r}$

$$\phi = x^2 + y^2$$

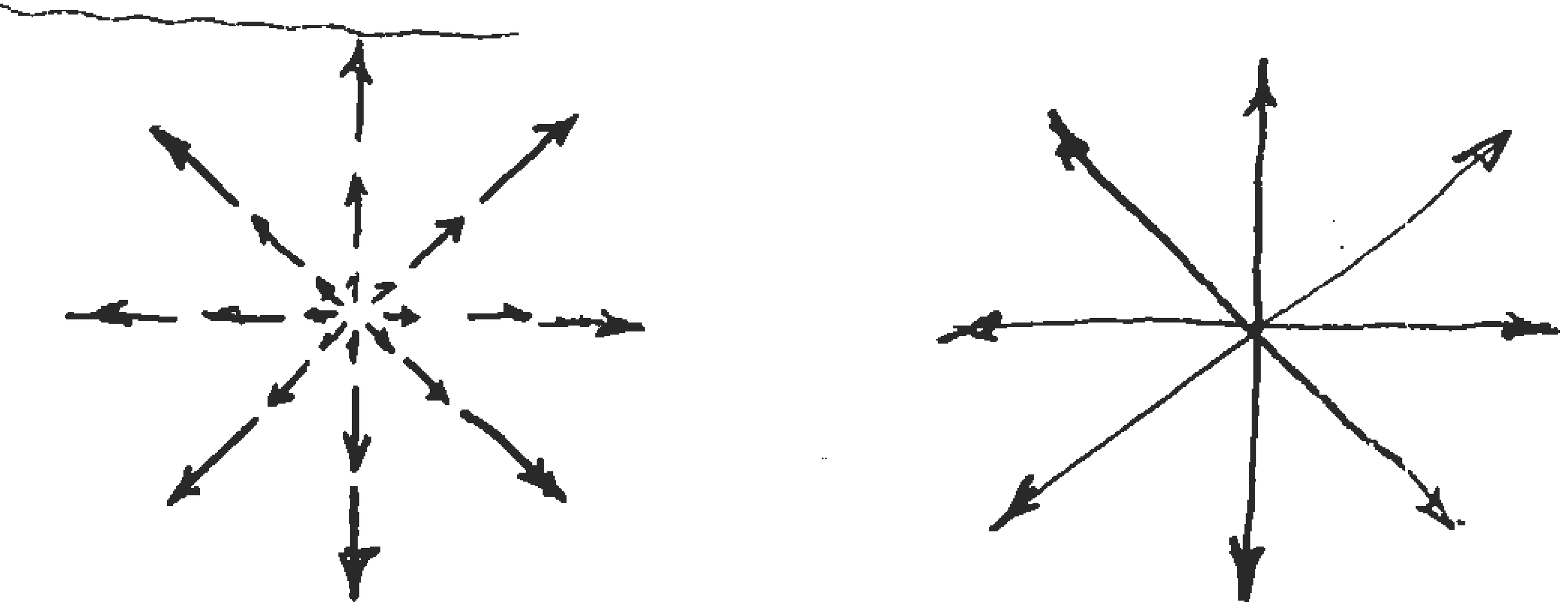
$$u = \frac{\partial \phi}{\partial x} = 2x$$

$$v = \frac{\partial \phi}{\partial y} = 2y$$

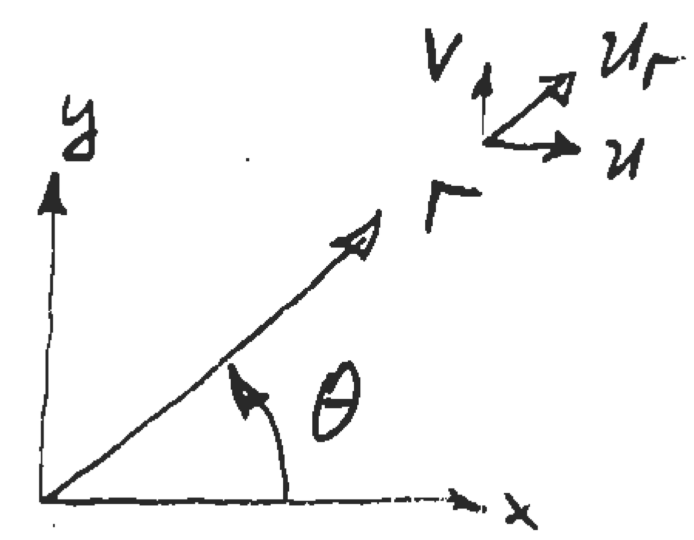
Streamlines: $\frac{dy}{dx} = \frac{v}{u} = \frac{y}{x}$

$$\frac{dy}{y} = \frac{dx}{x}$$

$$y = Cx$$



$|\vec{V}|$ varies as r

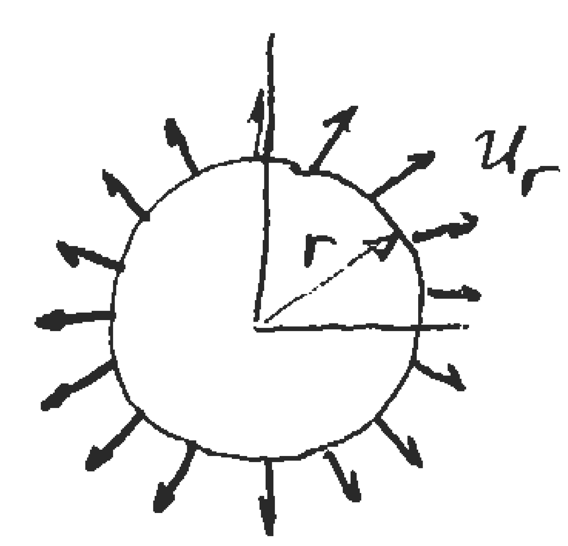


b) For $\psi = \arctan\frac{y}{x}$: $u = \frac{x}{r^2} = \frac{\cos\theta}{r}$

$$v = \frac{y}{r^2} = \frac{\sin\theta}{r}$$

Radial velocity: $u_r = u\cos\theta + v\sin\theta = \frac{1}{r}$

Volume flow rate: $\dot{V} = 2\pi r u_r = 2\pi$ (constant)



For $\phi = x^2+y^2$: $u = 2x = 2r\cos\theta$

$$v = 2y = 2r\sin\theta$$

Radial velocity: $u_r = u\cos\theta + v\sin\theta = 2r$

Volume flow rate: $\dot{V} = 2\pi r u_r = 4\pi r^2$ (increases as r^2)

c) $\phi = x^2+y^2$ is not feasible to set up, since $\nabla \cdot \vec{V} \neq 0$ for this flow, so it doesn't obey mass conservation in a low speed flow situation. Lack of mass conservation is further evidenced by $\dot{V}$ increasing with r. Mass is being created "out of thin air" (pun intended)